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Audrey Terras
University of California, San Diego

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To the Julias.

. . . Sometimes when I am alone in the dark, and the
universe reveals yet another secret, I say the names
of my long lost sisters, forgotten in the books that
record our science –
Aglaonice of Thessaly,
Hypatia,
Hildegard,
Catherine Hevelius,
Maria Agnesi
– as if the stars themselves could

remember . . . .

Caroline Herschel (1740–1848) by Siv Cedering
in *Math. Intelligencer.* Vol. 7, No. 4 (1985), p. 72

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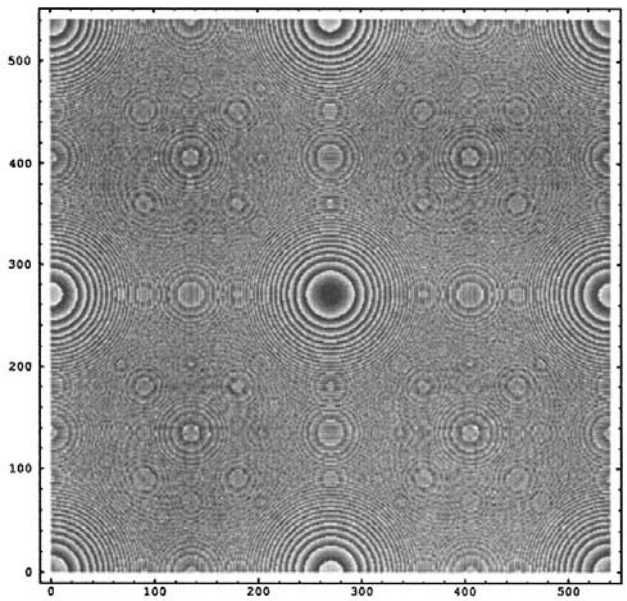
Preface

My thanks go to lots of creatures – many of them at USCD. In particular, I want to mention MSRI for supporting me during several productive visits, my graduate and undergraduate students at USCD who helped with various versions of this book, and the ever patient (NOT!) POSSLQ. I particularly want to thank Aubin Whitley for help with the proof reading and indexing. And I would like to thank Ellen Tirpak and the staff at TechBooks as well as Alan Harvey at Cambridge University Press. Thanks also to the AMS, DIMACS, and IMA for sponsoring some inspirational conferences. I also must thank the authors of the various computer programs I used – beginning with



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