

*Contents*

	<i>Preface</i>	xi
<b>1</b>	<b>A few tools from probability theory</b>	1
	1 Introduction	1
	2 The basic notions	2
	3 Distribution and similarity	3
	4 Product probability space	4
	5 The standard model; independence; Steinhaus and Rademacher sequences	4
	6 Integration: the main tools	5
	7 Symmetric random vectors	8
	8 Random functions and analytic sets	9
<b>2</b>	<b>Random series in a Banach space</b>	11
	1 Introduction	11
	2 Summability methods	12
	3 Sums of symmetric random vectors; two lemmas	14
	4 Proof of theorem 1	15
	5 Rademacher series $\sum_1^\infty \pm u_n$	18
	6 A principle of contraction	20
	7 The strong integrability for Rademacher series	23
	8 Exercises	25
<b>3</b>	<b>Random series in a Hilbert space</b>	28
	1 Introduction	28
	2 The Kolmogorov inequality	29
	3 The Paley–Zygmund inequalities	30
	4 Positive random series	32
		v

vi	<i>Contents</i>	
	5 Necessary and sufficient conditions for convergence and boundedness	33
	6 Exercises	35
<b>4</b>	<b>Random Taylor series</b>	37
	1 Introduction	37
	2 Singular points	38
	3 The symmetric case	39
	4 The general case	40
	5 Random Taylor series in two complex variables	41
	6 Random Dirichlet series	43
	7 Complements and exercises	44
<b>5</b>	<b>Random Fourier series</b>	46
	1 Introduction	46
	2 Auxiliary results on trigonometric series	47
	3 Rademacher series: the case $\sum_0^\infty x_n^2 = \infty$	49
	4 Rademacher series: the case $\sum_0^\infty x_n^2 < \infty$	51
	5 The general Paley–Zygmund theorem	53
	6 Auxiliary results on series of translates	54
	7 Convergence and boundedness in $C$ or $L^\infty$	56
	8 Convergence everywhere; the Billard theorem	58
	9 An application: Fourier coefficients of continuous functions	60
	10 Exercises	63
<b>6</b>	<b>A bound for random trigonometric polynomials and applications</b>	67
	1 Introduction	67
	2 Distribution of $M = \ P\ _\infty$	68
	3 Applications; a theorem of Littlewood and Salem; Sidon and Helson sets	70
	4 Another application: generalized almost periodic sequences	72
	5 Polynomials with unimodular coefficients	75
	6 Sums of sines	78
	7 Exercises	81
<b>7</b>	<b>Conditions on coefficients for regularity</b>	83
	1 Introduction	83
	2 A sufficient condition for $(1) \in C$	84
	3 Estimates for the modulus of continuity (subgaussian case)	86

<i>Contents</i>		vii
	4 A sufficient condition for $(1) \in A_\alpha$	88
	5 An application	90
	6 Exercises	91
<b>8</b>	<b>Conditions on coefficients for irregularity</b>	93
	1 Introduction	93
	2 Unboundedness: the Paley–Zygmund approach	94
	3 Unboundedness: a particular case	96
	4 Unboundedness: the general case	98
	5 Irregularity almost everywhere	99
	6 Irregularity everywhere	101
	7 Simultaneous inequalities	103
	8 Irregularity everywhere (continued)	104
	9 Divergence everywhere	106
	10 Exercises	108
<b>9</b>	<b>Random point-masses on the circle</b>	109
	1 Introduction	109
	2 Two theorems on Fourier–Stieltjes series	110
	3 Proof of theorem 2	112
	4 An almost everywhere divergent Fourier series	116
	5 Poisson transform of $\sum_1^\infty \varepsilon_j m_j \delta_{\theta_j}$	118
	6 A theorem on conjugate harmonic functions	121
	7 More about the case $\sum_1^\infty m_j^2 = 1$	124
	8 Exercises	125
<b>10</b>	<b>A few geometric notions</b>	128
	1 Introduction	128
	2 Hausdorff measures and dimensions; Frostman’s lemma	129
	3 Energy and capacity; Frostman’s theorem	132
	4 $\varepsilon$ -covering numbers	134
	5 Helices	135
	6 Quasi-helices; von Koch and Assouad curves	137
	7 More on dimensions	139
	8 Exercises	141
<b>11</b>	<b>Random translates and covering</b>	143
	1 Introduction	143
	2 Covering the circle: a sufficient condition	144
	3 Covering the circle: a necessary condition	149
	4 Covering the circle: the necessary and sufficient condition	150

viii	<i>Contents</i>	
	5 Covering a subset of $\mathbb{T}^q$ by random sets: a necessary	153
	6 Covering a subset of $\mathbb{T}^q$ : a sufficient condition; the case of convex $g_n$	156
	7 The case of non-flattening convex $g_n$ ; covering a set of given Hausdorff dimension	158
	8 The case of non-flattening convex $g_n$ (continued); dimension of the non-covered set	159
	9 Concluding remarks	161
	10 Exercises	162
<b>12</b>	<b>Gaussian variables and gaussian series</b>	165
	1 Introduction	165
	2 Formulas on Fourier transforms	166
	3 Gaussian random variables	168
	4 Some more formulas	171
	5 Around the Borel–Cantelli lemma	172
	6 Transient and recurrent gaussian series	173
	7 Gaussian series in a Banach space	175
	8 Exercises	177
<b>13</b>	<b>Gaussian Taylor series</b>	178
	1 Introduction	178
	2 A review of previous results	179
	3 The range of $F(z)$ ( $ z  < 1$ )	180
	4 The radial behavior: a recurrence condition	184
	5 The radial behavior: transience conditions	186
	6 Non-radial behavior: recurrence conditions	189
	7 Transience on circular sets	193
	8 Exercises	195
<b>14</b>	<b>Gaussian Fourier series</b>	197
	1 Introduction	197
	2 Review of known results	199
	3 Capacities and Hausdorff dimension reviewed	199
	4 Range of $F$	200
	5 The zeros of $F$	203
	6 A definition of $\delta^{(q)}(F)$	207
	7 The Malliavin theorem on spectral synthesis	209
	8 Exercises	210

	<i>Contents</i>	ix
<b>15</b>	<b>Boundedness and continuity for gaussian processes</b>	211
	1 Introduction	211
	2 Slepian's lemma	213
	3 Marcus and Shepp's theorem; the Pisier algebra	215
	4 Dudley's theorem	218
	5 Fernique's theorem	221
	6 Non-gaussian Fourier series	226
	7 Exercises	231
<b>16</b>	<b>The brownian motion</b>	233
	1 Introduction	233
	2 The Wiener process	233
	3 The Fourier–Wiener series	235
	4 More on local properties	237
	5 Stopping times, polar sets and newtonian capacity	242
	6 Self-crossing	245
<b>17</b>	<b>Brownian images in harmonic analysis</b>	250
	1 Introduction	250
	2 Brownian images	251
	3 Brownian image of a measure; proof of theorem 1	253
	4 Arithmetical properties of brownian images; proof of theorem 2	255
	5 Image of a measure by a gaussian Fourier series	257
	6 A construction of H. Cartan; proof of lemma 6	258
	7 A generalization of theorems 1 and 2	260
	8 Exercises	261
<b>18</b>	<b>Fractional brownian images and level sets</b>	263
	1 Introduction	263
	2 The gaussian processes $(n, d, \gamma)$	264
	3 Fractional brownian image of a measure; new Salem sets	265
	4 Fractional brownian images (continued); occupation density	267
	5 Level sets	272
	6 Uniqueness and continuity of $\delta(X - x)$	275
	7 Graphs	278
	8 Exercises	279
	<i>Notes</i>	281
	<i>Bibliography</i>	290
	<i>Index</i>	301