CAMBRIDGE TRACTS IN MATHEMATICS
General Editors
B. BOLLOBAS, F. KIRWAN, P. SARNAK, C.T.C. WALL

115

Introduction to H_p Spaces

Paul Koosis
McGill University in Montreal

Introduction to H_p Spaces

Second edition, corrected and augmented

With two appendices by

V. P. Havin
St. Petersburg (Leningrad) State and McGill Universities

CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo

Cambridge University Press
The Edinburgh Building, Cambridge CB2 2RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521455213

First edition published 1980
Second edition published 1998

A catalogue record for this publication is available from the British Library

ISBN-13 978-0-521-45521-3 hardback
ISBN-10 0-521-45521-9 hardback

Transferred to digital printing 2006

Contents

Preface to Second Edition

The first edition of this book was published in 1980 in the LMS Lecture Note Series, and a Russian translation by V.V. Peller and A.G. Tumarkin, made under the direction of V.P. Havin, the editor, appeared in 1984.

Both versions of the book are now out of print, and for the past couple of years people have been asking me how they might procure a copy of it. The Cambridge University Press has therefore decided to put out a second edition, and I am grateful to Dr. David Tranah, the Press' senior mathematics editor, for his having arranged to issue it in somewhat improved typographical format as a Cambridge Tract.

In preparing the first edition I had tried to make the exposition as accessible as I could by concentrating on what I thought were the main ideas in the subject rather than on including as many results as possible. The readers I had in mind were those with some training in analysis who were trying to gain a secure foothold in the theory of H_p spaces, whether with the aim of eventually doing serious work in that subject or for the purpose of understanding its applications in other areas (e.g. in operator theory – some of the material is now even used in electrical engineering). I have been guided by the same concern while working on the second edition and have for that reason tried to preserve the book's original character. That has especially meant refraining from attempting to turn it into what it was never intended to be – an all-encompassing treatise.

My first and main preoccupation has been to put to rights the first edition's many troublesome misprints, oversights and actual mistakes. I am grateful to the people who have called some of these to my attention, and their names are cited in the appropriate places. There were at least two serious errors in mathematical reasoning: at the end of §F in Chapter IV and in §G.1 of Chapter X. The proof of the lemma in Chapter II, §C.2, while not actually wrong, was certainly incomplete. An effort has been made to remedy these defects and several other less serious ones; I hope it has been successful.

Some new material that I consider really important has been added. Lindelöf's second theorem on conformal mapping can now be found in the new §C.3 of Chapter II. That result is one of the two main ingredients in the proof of Kellogg's theorem, included in the new §F of Chapter V. The simple geometric construction of Chapter VIII, §D.1 is used to obtain the atomic decomposition for $\Re H_1$ given in a new §E of that chapter, the old §E having become §F. This decomposition is then applied to give an alternative proof of the hard part of Fefferman's duality theorem in a new §G of Chapter X; the former §G is now §H.

The old appendix on Wolff's proof of the corona theorem is now Chapter XI. Where it used to stand there are two new appendices by V.P. Havin, on Peter Jones' interpolation formula and on Havin's proof of the weak sequential completeness of $L_1/H_1(0)$. The first edition of this book had already gone to press when I learned about the former topic, and time had not permitted my inclusion of the other one. Professor Havin was kind enough to write appendices on both for the Russian edition, and it is his appendices that are reproduced here, in my translation. These are included with his permission; he has read them and made certain suggestions that have been adopted. In making the translations I have tried to hew as closely as I could to Havin's own style, considerably different from mine.

The reader will probably also notice some differences in style between the new passages written by me and the older parts of the book. I cannot help that, for I am no longer able to write as I did in 1980.

I have been encouraged over the years by people with whom I was in large part unacquainted, who let me know in various ways that they liked one of the earlier versions of the book. I thank all of them for that encouragement, which strengthened my motivation to go on writing about mathematics.

Argenteuil County, Québec, near Boileau. October 10, 1993

Preface to First Edition

These are the lecture notes for a course I gave on the elementary theory of H_p spaces at the Stockholm Institute of Technology (tekniska högskolan) during the academic year 1977–78. The course concentrated almost exclusively on concrete aspects of the theory in its simplest cases; little time was spent on the more abstract general approach followed, for instance, in Gamelin's book. The idea was to give students knowing basic real and complex variable theory and a little functional analysis enough background to read current research papers about H_p spaces or on other work making use of their theory. For this reason, more attention was given to techniques and to what I believed were the ideas behind them than to the accumulation of a great number of results.

The lectures, about H_p spaces for the unit circle and the upper half plane, went far enough to include interpolation theory and *BMO*, but not as far as the corona theorem. That omission has, however, been put to rights in an appendix, thanks to T. Wolff's recent work. His proof of the corona theorem given there is a beautiful application of some of the methods developed for the study of *BMO*.

For Carleson's original proof of the corona theorem the reader may consult Duren's book. I have not included the more recent applications of the geometric construction Carleson devised for that proof, such as Ziskind's. Work of Douglas, Sarason, S-Y. Chang and Marshall on the algebras lying between H_∞ and L_∞ is not treated either.

Time did not allow me to cover the work of Hunt, Muckenhoupt and Wheeden on weighted mean value inequalities for harmonic conjugation. I did, however, give the proof of the Helson-Szegö theorem. Marshall's theorem (on the uniformly closed convex hull of the set of Blaschke products) is included although I did not lecture on it, and my lecture treatment of Lindelöf's theorem (on behaviour of the conformal mapping function near a point of tangency of the boundary) has been expanded.

In general, the notes stay quite close to the lectures as they were given. The style is loose and informal. Precise bibliographical references are not given in the text, nor the historical outlines at the end of each chapter that one has come to expect. A very partial bibliography is included; its purpose is to suggest further reading rather than to cover the subject thoroughly or to give due credit to all the workers in the field.

Topics not covered here, as well as the further ramifications of those I do cover, are treated in Garnett's extensive monograph now in the process of final revision. That book is recommended to the reader who wishes to go further.

I want to thank Harold Shapiro, Mats Essén and Magnus Giertz of the Stockholm Institute of Technology mathematics department for having helped me get an appointment to give this course. I want to thank the students and auditors for having successfully supported an extension of the course's length from the one semester originally planned to a full academic year. These were the students: Jockum Aniansson, Mats Lindberg, Lars Svensson and Anders Östrand. Björn Gustafsson audited most of the lectures and Dr. Stormark attended many. I was honoured by Dr. G.O. Thorin's presence at all of them. To all these people, my best wishes and warmest regards.

Los Angeles May 26, 1979