Cambridge University Press
978-0-521-45506-0 - Gravity from the Ground Up
Bernard Schutz
Excerpt
More information

1

Gravity on Earth:
the inescapable force

In this chapter: the simplest observations about gravity – it is universal and attractive, and it affects all bodies in the same way – have the deepest consequences. Galileo, the first modern physicist, founded the equivalence principle on them; this will guide us throughout the book, including to black holes. Galileo also introduced the principle of relativity, used later by Einstein. We begin here our use of computer programs for solving the equations for moving bodies.

▷Remember, terms in **boldface** are in the glossary.

▷The picture underlying the text on this page is of the famous bell tower at Pisa, where Galileo is said to have demonstrated the key to understanding gravity, that all bodies fall at the same rate. We will discuss this below. Photo by the author.

Gravity is everywhere. No matter where you go, you can't seem to escape it. Pick up a stone and feel its weight. Then carry it inside a building and feel its weight again: there won't be any difference. Take the stone into a car and speed along at 100 miles per hour on a smooth road: again there won't be any noticeable change in the stone's weight. Take the stone into the gondola of a hot-air balloon that is hovering above the Earth. The balloon may be lighter than air, but the stone weighs just as much as before.

This inescapability of gravity makes it different from all other forces of nature. Try taking a portable radio into a metal enclosure, like a car, and see what happens to its ability to pick up radio stations: it gets seriously worse. Radio waves are one aspect of the *electromagnetic force*, which in other guises gives us static electricity and **magnetic fields**. This force does not penetrate everywhere. It can be excluded from regions if we choose the right material for the walls. Not so for gravity. We could build a room with walls as thick as an Egyptian pyramid and made of any exotic material we choose, and yet the Earth's gravity would be right there inside, as strong as ever. *Gravity acts on everything the same way.*

Every body falls *toward* the ground, regardless of its composition. We know of no substance that accelerates *upwards* because of the Earth's gravity. Again this distinguishes gravity from all the other fundamental forces of Nature. **Electric charges** come in two different signs, the "+" and "-" signs on a battery. A negative **electron** attracts a positive **proton** but repels other electrons.

> There is a simple home experiment that will show this. If you have a clothes dryer, find a shirt to which a couple of socks are clinging after they have been dried. Pulling the socks off separates some of the charges of the molecules of the fabric, so that the charges on the sock will attract their opposites on the shirt if they are held near enough. But the socks have the same charge and repel each other when brought together.

The existence of *two* signs of electric charge is responsible for the shape of our everyday world. For example, the balance between attraction and repulsion among the different charges that make up, say, a piece of wood gives it rigidity: try to stretch it and the electrons resist being pulled away from the protons; try to compress it and the electrons resist being squashed up against other electrons. Gravity allows no such fine balances, and we shall see that this means that bodies in which gravity plays a dominant role cannot be rigid. Instead of achieving equilibrium, they have a strong tendency to collapse, sometimes even to **black holes**.

These two facts about gravity, that it is ever-present and always attractive, might make it easy to take it for granted. It seems to be just part of the background, a constant and rather boring feature of our world. But nothing could be further from the truth. Precisely because it penetrates everywhere and cannot be cancelled out, it is the engine of the Universe. All the unexpected and exciting discoveries of modern astronomy – **quasars**, **pulsars**, **neutron stars**, black holes – owe their existence to gravity. It binds together the gases of a **star**, the stars of a **galaxy**, and even galaxies into **galaxy clusters**. It has governed the formation of stars and it regulates the way stars create **chemical elements** of which we are made. On a grand scale, it controls the **expansion of the Universe**. Nearer to home, it holds planets in orbit about the Sun and satellites about the Earth.

The study of gravity, therefore, is in a very real sense the study of practically everything from the surface of the Earth out to the edge of the Universe. But it is even more: it is the study of our own history and evolution right back to the **Big Bang**. Because gravity is everywhere, our study of gravity in this book will take us everywhere, as far away in distance and as far back in time as we have scientific evidence to guide us.

Galileo: the beginnings of the science of gravity

In this section: Galileo laid the foundations for the scientific study of gravity. His demonstration that the speed of fall is independent of the weight of an object was the first statement of the principle of equivalence, which will lead us later to the idea of black holes.

We will begin our study of gravity with our feet firmly on the ground, by meeting a man who might fairly be called the founder of modern science: Galileo Galilei (1564–1642).

In Galileo's time there was a strong interest in the trajectories of cannonballs. It was, after all, a matter of life and death: an army that could judge how far gravity would allow a cannonball to fly would be better equipped to win a battle over a less well-informed enemy. Galileo's studies of the trajectory problem went far beyond those of any previous investigator. He made observations in the field and then performed careful experiments in the laboratory. These experiments are a model of care and attention to detail. He found out two things that startled many people in his day and that remain cornerstones of the science of gravity.

> First, Galileo found that the rate at which a body falls does not depend upon its weight. Second, he measured the rate at which bodies fall and found that their acceleration is constant, independent of time.

Figure 1.1. Galileo Galilei moved science away from speculation and philosophy and toward its modern form, insisting on the pre-eminence of careful experiment and observation. He also introduced the idea of describing the laws of nature mathematically. Meeting strong religious opposition in his native Italy, his ideas stimulated the growth of science in northern Europe in the decades after his death. Image reproduced courtesy of Mary Evans Picture Library.

After Galileo, gravity suddenly wasn't boring any more. Let's look at these two discoveries to find out why.

The story goes that Galileo took two iron balls, one much heavier than the other, to the top of the bell tower of Pisa and dropped them simultaneously. Most people of the day (and even many people today!) would probably have expected the heavier ball to have fallen much faster than the lighter one, but no: both balls reached the ground together.

The equality of the two balls' rates of fall went against the intuition and much of the common experience of the day. Doesn't a brick fall faster than a feather? Galileo pointed out that air resistance can't be neglected in the fall of a feather, and that to discover the properties of gravity alone we must experiment with dense bodies like stones or cannonballs, where the effects of air resistance are small. For such objects we find that speed is independent of weight.

But surely, one might object, we have to do much more work to lift a heavy stone than a light one, so doesn't this mean that a heavy stone "wants" to fall more than a light one and will do so faster, given the chance? No, said Galileo: weight has nothing to do with the speed of fall. We can prove that by measuring it. We have to accept the world the way we find it. This was the first step towards what we now call the *principle of equivalence*, which essentially asserts that gravity is indistinguishable from uniform accleration. We shall see that this principle has a remarkable number of consequences, from the weightlessness of astronauts to the possibility of black holes.

Investigation 1.1. Faster and faster: the meaning of uniform acceleration

In this investigation, we work out what Galileo's law of constant acceleration means for the speed of a falling body. The calculation is short, and it introduces us to the way we will use some mathematical symbols through the rest of the book.

We shall denote time by the letter t and the speed of the falling body by v (for *v*elocity). The speed at time t will be written $v(t)$. The acceleration of the body is g, and it is constant in time.

Suppose the body is dropped from rest at time $t = 0$. Then its initial speed is $v = 0$ at time $t = 0$, in other words $v(0) = 0$. What will be its speed a short time later?

Let us call this later time Δt. Here we meet an important new notation: the symbol Δ will always mean "a change in" whatever symbol follows it. Thus, a change in time is Δt. Similarly, we shall call the change in speed produced by gravity Δv. Normally we shall use this notation to denote small changes; here, for example, I have defined Δt to be "a short time later". We shall ask below how small Δt has to be in order to be "short".

The acceleration g is the change in the speed per unit time. This definition can be written algebraically as

$$g = \frac{\Delta v}{\Delta t}. \tag{1.1}$$

By multiplying through by the denominator of the fraction, we can solve for the change in speed:

$$\Delta v = g\Delta t. \tag{1.2}$$

Equation 1.1 basically defines g to be the *average* acceleration during the time Δt. If we take Δt to be very small, then this gives what we generally call the *instantaneous* acceleration. In this sense, "small" effectively means "as small as we can measure". If I have a clock which can reliably measure time accurate to a millisecond, then I would take Δt to be 1 ms if I wanted the instantaneous acceleration.

Now, Galileo tells us that the acceleration of a falling body does not in fact change with time. That means that the average acceleration during *any* period of time is the same as the instantaneous acceleration g. So in this particular case, it does not actually matter if Δt is small or not: Equation 1.1 is exactly true for any size of Δt. If we let t be any time, then we can rewrite Equation 1.2 as

$$\Delta v = gt.$$

We assumed above that the body was dropped from rest at time $t = 0$. This means that the initial speed is zero, and so the speed at a later time is just equal to Δv as given above. But if the body has an initial downward speed $v(0) = v_0$, then its subsequent acceleration only adds to the speed. This means that

$$v(t) = v(0) + \Delta v,$$

or

$$v(t) = gt + v_0. \tag{1.3}$$

Exercise 1.1.1: *Speed of a falling body*

Using the fact that the acceleration of gravity on Earth is $g = 9.8\ \mathrm{m\ s^{-2}}$, calculate the speed a ball would have after falling for two seconds, if dropped from rest. Calculate its speed if it were thrown downwards with an initial speed of $10\ \mathrm{m\ s^{-1}}$. Calculate its speed if it were initially thrown *upwards* with a speed of $10\ \mathrm{m\ s^{-1}}$. Is it falling or still rising after 2 s?

The acceleration of gravity is uniform

In this section: near the Earth, bodies accelerate downwards at a uniform rate.

Galileo performed a number of ingenious experiments with the rather crude clocks available in his day to demonstrate that the acceleration of falling objects is constant. Now, the acceleration of an object is the rate of change of its speed, so if the acceleration is constant then the speed changes at a constant rate; during any given single second of time, the speed increases by a fixed amount. We call this constant the **acceleration of gravity**, and denote it by g (for *g*ravity). Its value is roughly 9.8 meters per second per second. The units, meters per second per second, should be understood as "(meters per second) per second", giving the amount of speed (meters per second) picked up per second. These units may be abbreviated as m/s/s, but it is more conventional (and avoids the ambiguous† ordering of division signs) to write them as $\mathrm{m\ s^{-2}}$.

As with any physical law, there is no reason "why" the world had to be this way: the experiment might have shown that the speed increased uniformly with the distance fallen. But that is not how our world is made. What Galileo found was that speed increased uniformly with time of fall.

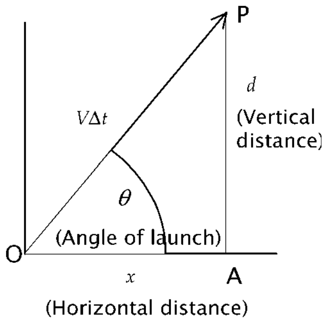

Figure 1.2. *For the calculation in Investigation 1.3 on page 5, the vertical and horizontal distances traveled by a cannonball launched at an angle θ are the sides of a right triangle whose hypotenuse is the total distance $V\Delta t$.*

We can find out what Galileo's law says about the distance fallen by doing our first calculations, Investigation 1.1 and 1.2. These calculations show that uniform acceleration implies that the speed a falling body gains is proportional to time and that the distance it falls increases as the square of the time. The calculation also has another purpose: it introduces the basic ideas and notation that we will use in later investigations to construct computer calculations of more complicated phe-

†Ambiguity: does m/s/s mean (m/s)/s or m/(s/s)? Either would be a valid interpretation of m/s/s, but in the second form the units for seconds cancel, which is not at all what is wanted.

Investigation 1.2. How the distance fallen grows with time

Here we shall calculate the distance $d(t)$ through which a falling body moves in the time t. Again we shall do this with simple algebra, but the ideas we use will lay the foundations for the computer programs we will write to solve harder problems later. Accordingly, much of the reasoning used below will be more general than is strictly necessary for the simple problem of a falling body.

We follow similar reasoning to that in Investigation 1.1 on the previous page. We are interested in the distance $d(t)$ fallen by the body by the time t. During the first small interval of time Δt, the body falls a distance Δd. (Our Δ notation again.) The *average* speed in this time is, therefore,

$$v_{avg} = \frac{\Delta d}{\Delta t}.$$

Solving this for Δd gives

$$\Delta d = v_{\text{avg}}\Delta t. \tag{1.4}$$

Now, we saw in Investigation 1.1 that the speed of the body changes during this interval of time. It starts out as zero (in the simplest case we considered) and increases to $g\Delta t$. So it seems to be an obvious guess that the average speed to use in Equation 1.4 is the average of these two numbers:

$$v_{avg} = \tfrac{1}{2}\,(g\Delta t + 0) = \tfrac{1}{2}g\Delta t.$$

If we put this into Equation 1.4, we find

$$\Delta d = \left(\tfrac{1}{2}g\Delta t\right)(\Delta t) = \tfrac{1}{2}g\,(\Delta t)^2\,. \tag{1.5}$$

I have said "an obvious guess" because it might not be right. If the acceleration of the body were a very complicated changing function of time, then its average speed over a time Δt might not be the average of its speeds at the beginning and end of the time-interval. For example, for some kinds of non-uniform acceleration it might happen that the body was at rest at the beginning and end of the interval, but not in between. Then its average speed might be positive, even though our guess would give zero.

Our guess is really only a good approximation in general if we choose the time-interval Δt small enough that the body's acceleration does not change by much during the interval. This gives a new insight into what is meant by a short time-interval: it must be short enough that the body's acceleration does not change by very much.

Of course, in the case of a falling body, the acceleration is constant, so we can expect Equation 1.5 to be *exact* for any time-interval, no matter how long. So if we replace Δt by t and Δd by $d(t)$, we find

$$d(t) = \tfrac{1}{2}gt^2. \tag{1.6}$$

Now suppose the body initially had a speed v_0. Then the average speed during the time Δt would be $v_0 + g\Delta t/2$, so Equation 1.5 would become

$$\Delta d = \left(v_0 + \tfrac{1}{2}g\Delta t\right)\Delta t = v_0\Delta t + \tfrac{1}{2}g\,(\Delta t)^2\,.$$

Then, if the body does not start at distance $d = 0$ but rather at distance $d(0) = d_0$, we have that $d(t)$ at a later time is

$$d(t) = \tfrac{1}{2}gt^2 + v_0\,t + d_0. \tag{1.7}$$

This is the full law of distance for a uniformly accelerating body.

The calculation we have just done may seem long-winded, especially to readers who are comfortable with calculus, because the operations I have gone through may seem like a beginner's introduction to calculus. This is not my aim, however. It will become clear in future examples that what we have actually met here is a method of doing calculations by **finite differences**; this method is at the heart of most computer calculations of the predictions of physical laws, and we will see that it will help us to solve much more difficult problems involving the motion of bodies under the influence of gravity. We can use finite differences reliably provided we use intervals of time that are short enough that the acceleration of a body does not change by much during the interval.

Exercise 1.2.1: *Distance fallen by a body*

For the falling ball in Exercise 1.1.1 on the preceding page, calculate the distance the ball falls in each of the cases posed in that exercise.

nomena. Anyone who can do algebra can follow these investigations.

Trajectories of cannonballs

In this section: Galileo introduced the idea that the horizontal and vertical motions of a body can be treated separately: the vertical acceleration of gravity does not change the horizontal speed of a body.

We can now take up one of the subjects that contributed to the Renaissance interest in gravity, namely the motion of a cannonball. We have discovered that the vertical motion of the ball is governed by the law of constant acceleration. What about its horizontal motion? Here, too, Galileo had a fundamental insight. He argued that the two motions are *independent*.

Consider dropping a rubber ball in an airplane moving with a large horizontal speed. The rate at which the ball falls does not depend on how fast the plane is moving. Moreover, imagine an observer on the ground capable of watching the ball: it keeps moving horizontally at the same speed as the plane even though it is free of any horizontal forces. That is, while it falls "straight down" relative to the passengers in the plane, it falls in an arc relative to the observer on the ground.

Let us transfer this reasoning to the example of a cannonball launched at an angle to the vertical so that its vertical speed is v_0 and its horizontal speed is u_0. Since there are no horizontal forces acting on the ball if we neglect air resistance, it will keep its horizontal speed as it climbs and falls, and the time it spends in the air will be the same as that of a ball launched vertically with the same speed v_0. Galileo showed that the trajectory that results from this is a parabola.

This would be easy for us to show, as well, by doing a little algebra.

Cambridge University Press
978-0-521-45506-0 - Gravity from the Ground Up
Bernard Schutz
Excerpt
More information

Investigation 1.3. The flight of the cannonball

Here we show how the finite-differences reasoning of the two previous investigations allows us to construct a computer program to calculate the flight of a cannonball, at least within the approximation that the ball is not affected by air resistance.

From this book's website you can download listing of the Java program `CannonTrajectory`. If you download the `Triana` software as well you can run the program and compute the trajectory of a cannonball fired at any given initial speed and at any angle. Figure 1.3 on the next page displays the result of the computer calculation for three trajectories, all launched with the same speed at three different angles. (The Triana software will produce plots of these trajectories. The figures produced for this book have, however, been produced by more sophisticated scientific graphics software.)

Here is how the program is designed. The idea is to calculate the body's horizontal and vertical position and speed at successive times spaced Δt apart. Let d be the vertical position and x the horizontal one, both zero to start. If the ball is launched with speed V at an angle θ with the horizontal (as in Figure 1.2 on page 3), then our first job is to deduce the vertical and horizontal speeds, which Galileo showed behaved independently of one another after launch.

Suppose that we turn off gravity for a moment and just watch a cannonball launched with speed V at an angle θ to the ground. Then after a small time Δt, it has moved a distance $V\Delta t$ in its launch direction. This is the distance OP in Figure 1.2. Simple trigonometry tells us that this distance is the **hypotenuse** of a right triangle whose other sides are the lines PA (the vertical distance d it has traveled) and AO (the horizontal distance x it has traveled). Then by definition we have

$$\sin\theta = \frac{d}{V\Delta t} \quad \Rightarrow \quad d = V\Delta t \sin\theta,$$

$$\cos\theta = \frac{x}{V\Delta t} \quad \Rightarrow \quad x = V\Delta t \cos\theta.$$

The vertical speed is the vertical distance d divided by the time Δt, and similarly for the horizontal speed. We therefore find that the initial vertical speed is

$$v_0 = V\sin\theta$$

and the initial horizontal speed is

$$u_0 = V\cos\theta.$$

The horizontal speed remains fixed, so the horizontal distance increases by $u_0 \times \Delta t$ each time step. The vertical speed decreases by $g\Delta t$ each time step, and we calculate the vertical distance using the *average* vertical speed in each time step. (In vertical motion, upward speeds are positive and downward ones negative.) The program sets up a **loop** to calculate the variables at successive time-steps separated by a small amount of time.

Normally one would expect a program like this to become more accurate for smaller time-steps, because of the remark we made in Investigation 1.2: our method of taking finite steps in time is better if the acceleration is nearly constant over a time step. In this case the relevant time step is Δt. By making Δt sufficiently small, one can always insure that the acceleration changes by very little during that time, and therefore that the accuracy of the program will increase. But in the present case that does not happen because our method of using the average speed over the time-step gives the *exact* result for uniform acceleration.

Let us look at the results of the three calculations in Figure 1.3 on the following page. Of these, the trajectory with the largest range for a given initial speed is the one that leaves the ground at a 45° angle. In fact it is not hard to show that this trajectory has the largest range of all possible ones. What is this range? We could calculate it from the results of Investigation 1.2, but in the spirit of our approach we shall try to guess it from the numerical calculation.

Given that the initial angle will be 45°, the range can only depend on the initial speed V and the acceleration g. The range is measured in meters, and the only combination of V and g that has the units of length is V^2/g : $(\mathrm{m\,s^{-1}})^2/(\mathrm{m\,s^{-2}}) = \mathrm{m}$. We therefore can conclude that there is some constant number b for which $range = bV^2/g$. (This reasoning is an example of a powerful technique called **dimensional analysis**, because one is trying to learn as much as possible from the units, or **dimensions**, of the quantities involved in the problem.)

The numerical results let us determine b. Since the calculation used $V = 100\,\mathrm{m\,s^{-1}}$, it follows that V^2/g is 1020 m. From the graph the range looks like 1020 m as well, as nearly as I can estimate it. Since the value of b is likely to be simple, it almost surely equals 1. An algebraic calculation shows this to be correct:

$$\text{maximum range} = V^2/g.$$

The reader is encouraged to re-run the program with various initial values of V to check this result.

Exercise 1.3.1: *Small steps in speed and distance*

Suppose that at the n^{th} time-step t_n, the vertical speed is v_n and the vertical distance above the ground is h_n. Show that at the next time-step $t_{n+1} = t_n + \Delta t$, the vertical speed is $v_{n+1} = v_n - g\Delta t$. Using our method of approximating the distance traveled by using the average speed over the interval, show that at the next time-step the height will be

$$h_{n+1} = h_n + \tfrac{1}{2}(v_n + v_{n+1})\Delta t = h_n + v_n\Delta t - \tfrac{1}{2}g(\Delta t)^2.$$

Exercise 1.3.2: *Suicide shot*

What is the *minimum* range of a cannonball fired with a given speed V, and at what angle should it be aimed in order to achieve this minimum?

Exercise 1.3.3: *Maximum range by algebra*

For readers interested in verifying the guess we made above from the numerical data, here is how to calculate the range at 45° algebraically. The range is limited by the amount of time the cannonball stays in the air. Fired at 45° with speed V, how long does it take to reach its maximum height, which is where its vertical speed goes to zero? Then how long does it take to return to the ground? What is the total time in the air? How far does it go horizontally during this time? This is the maximum range.

Exercise 1.3.4: *Best angle of fire*

Prove that 45° is the firing angle that gives the longest range by calculating the range for any angle and then finding what angle makes it a maximum. Use the same method as in Exercise 1.3.3.

But instead we show in Investigation 1.3 on the previous page how to use a personal computer to calculate the actual trajectory of a cannonball. These computer techniques will form the foundation of computer programs later in this book that will calculate other trajectories, such as planets around the Sun, stars in collision with one another, and particles falling into black holes.

Galileo: the first relativist

In this section: Galileo introduced what we now call the principle of relativity, which Einstein used as a cornerstone of his own revolutionary theories of motion and gravity almost 300 years later.

It would be hard to overstate Galileo's influence on science and therefore on the development of human society in general. He founded the science of **mechanics**; his experiments led the English scientist Isaac Newton (1642–1726) to discover his famous laws of motion, which provided the foundations for almost all of physics for 200 years. And almost 300 years after his death his influence was just as strong on Albert Einstein. The German–Swiss physicist Einstein (1879–1955) replaced Newton's laws of motion and of gravity with new ones, based on his theory of relativity. Einstein's revolutionary theories led to black holes, the Big Bang, and many other profound predictions that we will study in the course of this book. Yet Einstein, too, kept remarkably close to Galileo's vision.

The main reason for Galileo's influence on Einstein is that he gave us the first version of what we now call the **principle of relativity**. We have already encountered Galileo's version: the vertical motion of a ball does not depend on its horizontal speed, and its horizontal speed will not change unless a horizontal force is applied.

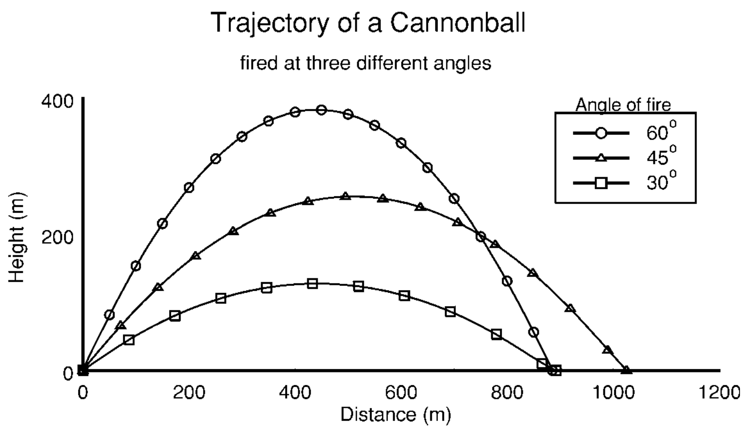

Figure 1.3. Trajectories computed by the program developed in Investigation 1.3 on the previous page, for three angles of firing, each at the same initial speed of 100 m s^{-1}. The trajectory at 45° goes furthest.

Where we used a fast-flying airplane to justify this, Galileo imagined a sailing ship on a smooth sea, but the conclusion was the same: an experimenter moving horizontally will measure the same acceleration of gravity in the vertical direction as he would if he were at rest.

Galileo took this idea and drew a much more profound conclusion from it. The radical proposal made half a century earlier by the Polish priest and astronomer Nicolas Copernicus (1473–1543), that the Earth and other planets actually moved around the Sun (see Figure 1.4), was still far from being accepted by most intellectuals in Galileo's time. Although the proposal explained the apparent motions of the planets in a simple way, it was open to an important objection: if the Earth is moving at such a rapid rate, why don't we *feel* it? Why isn't the air left behind, why doesn't a ball thrown vertically fall behind the moving Earth?

Galileo used the independence of different motions to dispose of this objection. Galileo's answer is that a traveler in the cabin of a ship on a smooth sea also does not feel his ship's motion: all the objects in the cabin move along with it at constant speed, even if they are just resting on a table and not tied down. Anything that falls will fall vertically in the cabin, giving no hint of the ship's speed. So it is on the Earth, according to Galileo: the air, clouds, birds, trees, and all other objects all have the same speed, and this motion continues until something interferes with it. There is, in other words, no way to tell that the Earth is moving through space except to look at things far away, like the stars, and see that it is.

Figure 1.4. *The Copernican view of the planets known in Galileo's day as they orbit the Sun.*

Today we re-phrase and enlarge this idea to say that *all* the laws of physics are just the same to an experimenter who moves with a uniform motion in a straight line as they are to one who remains at rest, and we call this the *principle of relativity*. We shall encounter many of its consequences as we explore more of the faces of gravity.

Unfortunately for Galileo, his clear reasoning and his observations with one of the first telescopes made him so dangerous to the established view of the Roman Catholic Church that in his old age he was punished for his views, and forced to deny them publicly. Privately he continued to believe that the planets went around the Sun, because he had discovered with his telescope that the moons of Jupiter orbit Jupiter in the same way that the planets orbit the Sun.

> Today we recognize Galileo as the person who, more than anyone else, established the Copernican picture of the Solar System.

Figure 1.5. *Part of a sketch by Galileo of the positions of Jupiter (open circles) and its moons (stars) on a sequence of nights (dates given by the numbers). The big changes from night to night puzzled Galileo. At first he believed that Jupiter itself was moving erratically, but after a few observations he realized that the "stars" were moons orbiting Jupiter in the same way that the planets orbit the Sun.*

2

And then came Newton: gravity takes center stage

In this chapter: we learn about Newton's postulate, that a single law of gravity, in which all bodies attract all others, could explain all the planetary motions known in Newton's day. We also learn about Newton's systematic explanation of the relationship between force and motion. When we couple this with Galileo's equivalence principle, we learn how gravity makes time slow down.

Born in the same year, 1642, as Galileo died, Isaac Newton revolutionized the study of what we now call physics. Part of his importance comes from the wide range of subjects in which he made fundamental advances – mechanics (the study of motion), optics, astronomy, mathematics (he invented calculus), ... – and part from his ability to put physical laws into mathematical form and, if necessary, to invent the mathematics he required. Although other brilliant thinkers made key contributions in his day – most notably the German scientist Gottfried Leibniz (1646–1716), who independently invented calculus – no physicist living between Galileo and Einstein rivals Newton's impact on the study of the natural world.

Nevertheless, it is hard to imagine that Newton could have made such progress in the study of motion and gravity if he had not had Galileo before him. Newton proposed three fundamental **laws of motion**. The first two are developed from ideas of Galileo that we have already looked at:

> The *first law* is that, once a body is set in motion, it will remain moving at constant speed in a straight line unless a force acts on it. This is just like the rubber ball dropped inside the airplane of Chapter 1.

This is basically Galileo's idea, which led him to his principle of relativity, that motions in different directions could be treated independently. Notice that, since particles travel in *straight* lines unless disturbed, the directions along which motion is independent must also be along straight lines.

> Newton's *second law* is that, when a force is applied to a body, the resulting *acceleration* depends only on the force and on the mass of the body: the larger the force, the larger the acceleration; and the larger the mass of a body, the smaller its acceleration.

Figure 2.1. Brilliant and demanding, Isaac Newton created theoretical physics. Besides devising the laws of gravity and mechanics, he invented calculus, still the central mathematical tool of physicists today. (Original engraving by unknown artist, courtesy AIP Emilio Segrè Visual Archives, Physics Today Collection.)

This dependence of the acceleration a on the force F and the mass m can be written as an equation:

$$a = F/m.$$

It is more conventional to write it in the equivalent form

$$F = ma. \tag{2.1}$$

The second law: weight and mass

In this section: how force, mass, and acceleration are related to one another, and the difference between weight and mass.

The second law fits our everyday experience of what happens when we push something. If we have a heavy object on wheels (to allow us to ignore friction for a moment) and we give it a push, its speed increases (it accelerates) as long as we continue to push it. Then it moves along at a constant speed after we stop pushing. (Friction eventually slows it down, but that is just another force exerted by the surface it is moving across.) If we push it harder, it accelerates faster, so it is not unreasonable to guess that the acceleration might be proportional to the force we exert on it. Moreover, if we load more things on top of the object we are pushing, then to get the same acceleration, we need to push harder, so again we might guess that the force required would be proportional to the mass. Newton not only made these guesses, but he assumed (or hoped!) that the force would depend on nothing else besides the mass of the body and the acceleration produced.

Newton made an important distinction between two concepts that are often used interchangeably in everyday language: **mass** and **weight**. The mass of an object is, as we have just seen, the way it "resists" being accelerated. (Physicists sometimes call this its **inertia**.) The weight of an object is the force of gravity on it. When we step on our bathroom scales, we measure our weight; if we were to put the scales on the Moon, where gravity is weaker, we would get a lower reading. Our mass would not have changed, however. It would take the same force to accelerate us on a smooth horizontal track on the Moon as it would on the Earth.

▷A good illustration of how everyday language uses such terms differently from the way we use them in physics is provided by dieting. About twice a year, when I go on a diet, I tell my friends that I am trying to lose weight. Mercifully, none of them has yet pointed out that the surest way to lose weight is to go to the Moon! That would not really help, of course, since what I am really trying to do, for the sake of my health, is to lose *mass*. If I stay on the Earth, then of course losing weight implies losing mass.

The second law leads to a remarkable insight when combined with Galileo's discovery that bodies of different masses accelerate under gravity at the same rate: it tells us that a body's *weight* must be proportional to its *mass*. Here is the reasoning.

Suppose we lift a heavy body off the floor and hold it. What we *feel* as its weight is really the sensation of exerting an upwards force upon it to hold it against the force of gravity. When we exert this force on a body, it remains at rest in our hands. By the first law, we conclude that the total force on it is zero: our upwards force just cancels the downwards force of gravity on the object. Therefore the weight of the body *is* the force of gravity on it. If we now release the body, the force of gravity on it is the same but is no longer balanced by our hands' force. The body accelerates downwards: it falls.

But what *is* its acceleration in response to this force of gravity? Galileo observed that the acceleration of the body does not depend on its weight: it is the *same* for everything. Now, in Equation 2.1 on the previous page, the only way that we can change the force F (the weight) without changing the acceleration a is if we change the mass m in proportion to F: the force of gravity on a body is proportional to its mass.

Newton's reasoning here leads to an experimentally verifiable conclusion. Both mass and weight could be measured independently, the weight using scales and the mass by measuring the acceleration of the body in response to a given *horizontal* force and then using Equation 2.1 on the preceding page to infer its mass. If we divide the weight by the mass, we should get the acceleration of gravity. Put mathematically, this says that the force of gravity F_{grav} on any object equals its mass m times the acceleration of gravity g:

$$F_{\text{grav}} = mg. \tag{2.2}$$

In honor of Newton, scientists have agreed to measure force in units called *newtons*: one newton (N) of force equals 1 kg times an acceleration of $1\,\mathrm{m\,s^{-2}}$. The weight of a body in newtons is then just its mass in kilograms times the acceleration of gravity, $9.8\,\mathrm{m\,s^{-2}}$.

As we have noted, Equation 2.2 is experimentally verifiable: if it holds for any body, then this experiment serves as a test of the second law itself.

> Once this law of motion was checked experimentally, Newton's argument led to a reformulation of Galileo's *principle of equivalence*: the mass of a body (ratio of force to acceleration) is proportional to its weight. From this statement and Equation 2.1 on the preceding page,

Galileo's original observation that all bodies fall with the same acceleration follows.

This was the way two centuries of physicists thought of the principle of equivalence. It was strikingly confirmed by the Hungarian physicist Baron Roland von Eötvös (1848–1919) in 1889 and in 1908. In one of the most accurate physics experiments of his time, von Eötvös showed that many different materials fell with the same acceleration, to within a few parts in a billion!

We now know that this form of the equivalence principle applies not just to ordinary bodies, but also to bodies with the strongest of gravitational fields in general relativity, even to black holes. However, Einstein's general theory of relativity did change Newtonian mechanics in some respects, so the way that modern physicists think about the principle of equivalence is also rather different from the Newtonian form. We will have to wait a few pages before we take a look at the modern reformulation, because we have not by any means finished with Newton's work yet. He made two further landmark contributions to our subject: his third law of motion and his law of gravitation.

The third law, and its loophole

In this section: when you push on a body, it pushes back on you. Normally the two forces are the same size. But when bodies are separated, they can differ, and this leads to a force called radiation reaction.

Newton added another law, not explicit in Galileo's work, but which he needed to make the whole science of mechanics self-consistent.

> Newton's *third law* states that if I exert a force on an object, then it exerts a force back on me that is exactly equal in magnitude and opposite in direction to the one I have applied. This law is often paraphrased as "action equals reaction".

This law often strikes newcomers to the subject as contradictory: if there are two equal and opposite forces, don't they cancel? If the object I push on moves and I don't, doesn't that mean that I pushed harder on it than it pushed on me? These difficulties are always the result of mis-applying Newton's second law. Only the forces acting *on* an object contribute to its acceleration. The equal and opposite forces in the third law act on different objects, and so there is no way that they can cancel each other.

To see our way past such doubts with a concrete example, consider again the feeling of a weight in the hand. Suppose I hold an apple. I have to exert a force on the apple, equal to its weight, to keep it in one place. This force is exerted through my hand, but the hand doesn't stay where it is all by itself: it is kept there by the force exerted on it by my arm. (The tired feeling I eventually get in the muscles of my arm leaves me no room to doubt this!) Since the hand isn't moving but the arm is exerting a force on it, there must be a balancing force on it as well, and this can only come from the apple. So as I exert a force on the apple, it exerts a force back on me. How much of a force? Newton argued that this "reaction" by the apple must be equal to the force I exert on it.

There are several ways of seeing that this is reasonable. Suppose, for instance, that the force exerted by the apple on my hand was only half of its weight. What makes the hand special, that it gets back only half the force it gives out? Why wouldn't it be the other way around, that the apple should receive from my hand only half the force it exerts on my hand? This lack of symmetry, where the hand gets only half the force back that it exerts, while the apple gets twice its force back, makes the "half-reaction" law illogical.

> The third law has an important practical consequence: it is responsible for almost all propulsion. We walk by pushing backwards with our feet