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Arithmetic of Blowup Algebras

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Contents

Foreword

Commutative Algebra, flushed with its great successes in dealing with the difficult homological questions that originated in the 60's, has flourished recently around many clusters of related activities. To name a few, in no particular order of significance: linkage theory, Cohen–Macaulay approximation, theory of Hilbert functions, the structure of free resolutions, tight closure, and a growing web of relationships to computer algebra and combinatorics. All this, while strengthening its connections to algebraic geometry and other parts of algebra and retaining the vitality of its established domains.

This is a panorama that begs for systematic accounts of these developments to guide a possible reader through a bountiful but scattered literature. Several efforts have begun to address these needs and there are definite stirrings of an accelerating pace.

Another of these nuclei of activity is the theory and practice of the so-called Rees or blowup algebras. Through its many variants it interacts in some measure with all the areas listed above. Furthermore, it is generating questions at a much faster clip than they are being dealt with. It became then a likely target to be written about.

We decided to assemble some recent results and methods on these algebras in a manner which would be comprehensible/useful to a reader conversant with basic commutative algebra. It was the original intent to frame these notes into a second course in commutative algebra, requiring the core part of [192], along with a good foundation in homological algebra. The task made easier because so much of the technical background, particularly on local cohomology, is already dealt with in the excellent [103]. The reader would also be provided with several entry points along the text to make access to different parts of the notes more direct. For example, chapters 9 and 10 are nearly independent of the rest of the notes. Along the way, part of this intent decayed: The sheer task of bringing together and integrating properly the material became too demanding, and was replaced by the more modest aim of presenting some significant results and methods, while providing gateways to other developments, some even richer in content.

One early ground rule, the emphasis on bringing in the computer as a basic tool in most aspects of the discussion, did not change. In fact, it became clear that its role would have to be strengthened and even took a life of its own. A result has been the lengthy Chapter 10, with a potpourri of methods and recipes to deal with basic constructions in commutative algebra.

Foreword

These notes owe much to numerous conversations with many colleagues in the course of the past few years. We are particularly grateful to Jürgen Herzog, Craig Huneke, Aron Simis, Bernd Ulrich, and Rafael Villarreal; their published (and unpublished) work, lectures and comments influenced considerably the choice of topics. Several others, colleagues and friends, unsuspectingly will see their own words, in forgotten letters or messages, stare back at them! We are also thankful to Alberto Corso, Susan Morey, Claudia Polini, Maria Vaz Pinto and two anonymous reviewers for pointing out many inaccuracies in an early version of the notes and for offering helpful suggestions.

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