The Physics of Plasmas

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1.1 Introduction

The plasma state is often referred to as the *fourth* state of matter, an identification that resonates with the element of *fire*, which along with earth, water and air made up the elements of Greek cosmology according to Empedocles.† Fire may indeed result in a transition from the gaseous to the plasma state, in which a gas may be fully or, more likely, partially ionized. For the present we identify as *plasma* any state of matter that contains enough free charged particles for its dynamics to be dominated by electromagnetic forces. In practice quite modest degrees of ionization are sufficient for a gas to exhibit electromagnetic properties. Even at 0.1 per cent ionization a gas already has an electrical conductivity almost half the maximum possible, which is reached at about 1 per cent ionization.

The outer layers of the Sun and stars in general are made up of matter in an ionized state and from these regions winds blow through interstellar space contributing, along with stellar radiation, to the ionized state of the interstellar gas. Thus, much of the matter in the Universe exists in the plasma state. The Earth and its lower atmosphere is an exception, forming a plasma-free oasis in a plasma universe. The upper atmosphere on the other hand, stretching into the ionosphere and beyond to the magnetosphere, is rich in plasma effects.

Solar physics and in a wider sense cosmic electrodynamics make up one of the roots from which the physics of plasmas has grown; in particular, that part of the subject known as magnetohydrodynamics – MHD for short – was established largely through the work of Alfvén. A quite separate root developed from the physics of gas discharges, with glow discharges used as light sources and arcs as a means of cutting and welding metals. The word *plasma* was first used by Langmuir in 1928 to describe the ionized regions in gas discharges. These origins

† Empedocles, who lived in Sicily in the shadow of Mount Etna in the fifth century BC, was greatly exercised by fire. He died testing his theory of buoyancy by jumping into the volcano in 433BC.

are discernible even today though the emphasis has shifted. Much of the impetus for the development of plasma physics over the second half of the twentieth century came from research into controlled thermonuclear fusion on the one hand and astrophysical and space plasma phenomena on the other.

To a degree these links with ‘big science’ mask more bread-and-butter applications of plasma physics over a range of technologies. The use of plasmas as sources for energy-efficient lighting and for metal and waste recycling and their role in surface engineering through high-speed deposition and etching may seem prosaic by comparison with fusion and space science but these and other commercial applications have laid firm foundations for a new plasma technology. That said, our concern throughout this book will focus in the main on the physics of plasmas with illustrations drawn where appropriate from fusion and space applications.

1.2 Thermonuclear fusion

While thermonuclear fusion had been earlier indentified as the source of energy production in stars it was first discussed in detail by Bethe, and independently von Weizsäcker, in 1938. The chain of reactions proposed by Bethe, known as the carbon cycle, has the distinctive feature that after a sequence of thermonuclear burns involving nitrogen and oxygen, carbon is regenerated as an end product enabling the cycle to begin again. For stars with lower central temperatures the proton–proton cycle

$$\begin{aligned} {}_1\mathrm{H}^1 + {}_1\mathrm{H}^1 &\rightarrow {}_1\mathrm{D}^2 + \mathrm{e}^+ + \nu \qquad (1.44\,\mathrm{MeV}) \\ {}_1\mathrm{D}^2 + {}_1\mathrm{H}^1 &\rightarrow {}_2\mathrm{He}^3 + \gamma \qquad (5.49\,\mathrm{MeV}) \\ {}_2\mathrm{He}^3 + {}_2\mathrm{He}^3 &\rightarrow {}_2\mathrm{He}^4 + 2\,{}_1\mathrm{H}^1 \qquad (12.86\,\mathrm{MeV}) \end{aligned}$$

where e^+, ν and γ denote in turn a positron, neutrino and gamma-ray, is more important and is in fact the dominant reaction chain in lower main sequence stars (see Salpeter (1952)). Numbers in brackets denote the energy per reaction. In the first reaction in the cycle, the photon energy released following positron–electron annihilation (1.18 MeV) is included; the balance (0.26 MeV) carried by the neutrino escapes from the star. The third reaction in the cycle is only possible at temperatures above about 10^7 K but accounts for almost half of the total energy release of 26.2 MeV. The proton–proton cycle is dominant in the Sun, the transition to the carbon cycle taking place in stars of slightly higher mass. The energy produced not only ensures stellar stability against gravitational collapse but is the source of luminosity and indeed all aspects of the physics of the outer layers of stars.

The reaction that offers the best energetics for controlled thermonuclear fusion in the laboratory on the other hand is one in which nuclei of deuterium and tritium fuse to yield an alpha particle and a neutron:

$$ {}_1\mathrm{D}^2 + {}_1\mathrm{T}^3 \rightarrow {}_2\mathrm{He}^4 + {}_0\mathrm{n}^1 \quad (17.6\,\mathrm{MeV}) $$

The total energy output $\Delta E = 17.6\,\mathrm{MeV}$ is distributed between the alpha particle which has a kinetic energy of about 3.5 MeV and the neutron which carries the balance of the energy released. The alpha particle is confined by the magnetic field containing the plasma and used to heat the fuel, whereas the neutron escapes through the wall of the device and has to be contained by a neutron-absorbing blanket.

1.2.1 The Lawson criterion

Although the D–T reaction rate peaks at temperatures of the order of 100 keV it is not necessary for reacting nuclei to be as energetic as this, otherwise controlled thermonuclear fusion would be impracticable. Thanks to quantum tunnelling through the Coulomb barrier, the reaction rate for nuclei with energies of the order of 10 keV is sufficiently large for fusion to occur. A simple and widely used index of thermonuclear gain is provided by the Lawson criterion. For equal deuterium and tritium number densities, $n_{\mathrm{D}} = n_{\mathrm{T}} = n$, the thermonuclear power generated by a D–T reactor per unit volume is $P_{\mathrm{fus}} = \frac{1}{4} n^2 \langle \sigma v \rangle \Delta E$, where $\langle \sigma v \rangle$ denotes the reaction rate, σ being the collisional cross-section and v the relative velocity of colliding particles. For a D–T plasma at a temperature of 10 keV, $\langle \sigma v \rangle \sim 1.1 \times 10^{-22}\,\mathrm{m}^3\,\mathrm{s}^{-1}$ so that $P_{\mathrm{fus}} \sim 7.7 \times 10^{-35} n^2\,\mathrm{W\,m}^{-3}$. About 20% of this output is alpha particle kinetic energy which is available to sustain the fuel at thermonuclear reaction temperatures, the balance being carried by the neutrons which escape from the plasma. Thus the power absorbed by the plasma is $P_\alpha = \frac{1}{4} \langle \sigma v \rangle n^2 E_\alpha$ where $E_\alpha = 3.5\,\mathrm{MeV}$. This is the heat added to unit volume of plasma per unit time as a result of fusion.

We have to consider next the energy lost through radiation, in particular as bremsstrahlung from electron–ion collisions. We shall find in Chapter 9 that bremsstrahlung power loss from hot plasmas may be represented as $P_{\mathrm{b}} = \alpha n^2 T^{1/2}$, where α is a constant and T denotes the plasma temperature. Above some critical temperature the power absorbed through alpha particle heating outstrips the bremsstrahlung loss. Other energy losses besides bremsstrahlung have to be taken into consideration. In particular, heat will be lost to the wall surrounding the plasma at a rate $3nk_{\mathrm{B}}T/\tau$ where τ is the containment time and k_{B} is Boltzmann's constant. Balancing power gain against loss we arrive at a relation for $n\tau$. Lawson (1957) introduced an efficiency factor η to allow power available for heating to be expressed in terms of the total power leaving the plasma. The *Lawson criterion* for power

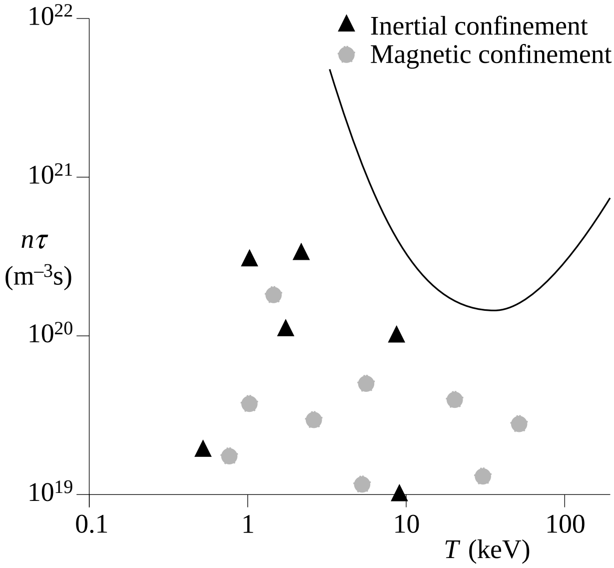

Fig. 1.1. The Lawson criterion for ignition of fusion reactions. Data points correspond to a range of magnetic and inertial confinement experiments showing a progression towards the Lawson curve.

gain is then

$$n\tau > \frac{3k_B T}{\left[\frac{\eta}{4(1-\eta)}\langle\sigma v\rangle\Delta E - \alpha T^{1/2}\right]} \tag{1.1}$$

This condition is represented in Fig. 1.1. Using Lawson's choice for $\eta = 1/3$ (which with hindsight is too optimistic), the power-gain condition reduces to $n\tau > 10^{20}\,\mathrm{m^{-3}\,s}$. The data points shown in Fig. 1.1 are $n\tau$ values from a range of both magnetically and inertially contained plasmas over a period of about two decades, showing the advances made in both confinement schemes towards the Lawson curve.

1.2.2 Plasma containment

Hot plasmas have to be kept from contact with walls so that from the outset magnetic fields have been used to contain plasma in controlled thermonuclear fusion experiments. Early devices such as Z-pinches, while containing and pinching the plasma radially, suffered serious end losses. Other approaches trapped the plasma in a magnetic bottle or used a closed toroidal vessel. Of the latter the *tokamak*, a contraction of the Russian for *toroidal magnetic chamber*, has been the most successful. Its success compared with competing toroidal containment schemes is

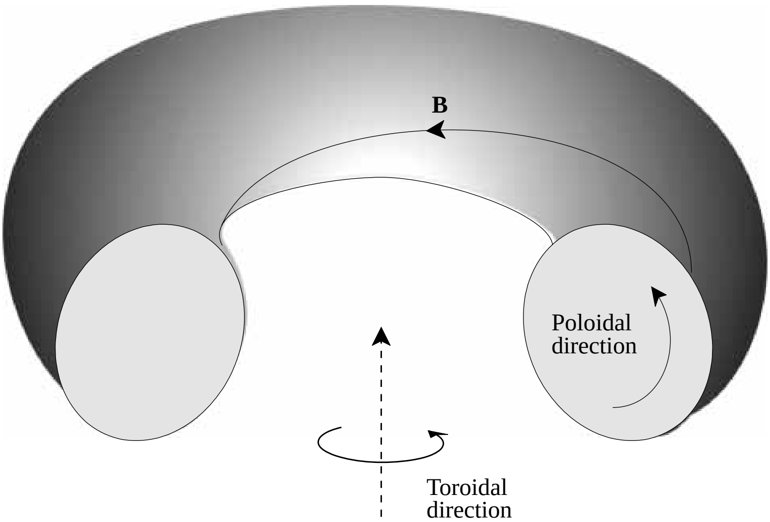

Fig. 1.2. Tokamak cross-section.

attributable in large part to the structure of the magnetic field used. Tokamak fields are made up of two components, one *toroidal*, the other *poloidal*, with the resultant field winding round the torus as illustrated in Fig. 1.2. The toroidal field produced by currents in external coils is typically an order of magnitude larger than the poloidal component and it is this aspect that endows tokamaks with their favourable stability characteristics. Whereas a plasma in a purely toroidal field drifts towards the outer wall, this drift may be countered by balancing the outward force with the magnetic pressure from a poloidal field, produced by currents in the plasma. Broadly speaking, the poloidal field maintains toroidal stability while the toroidal field provides radial stability. For a typical tokamak plasma density the Lawson criterion requires containment times of a few seconds.

Inertial confinement fusion (ICF) offers a distinct alternative to magnetic containment fusion (MCF). In ICF the plasma, formed by irradiating a target with high-power laser beams, is compressed to such high densities that the Lawson criterion can be met for confinement times many orders of magnitude smaller than those needed for MCF and short enough for the plasma to be confined inertially. The ideas behind inertial confinement are represented schematically in Fig. 1.3(a) showing a target, typically a few hundred micrometres in diameter filled with a D–T mixture, irradiated symmetrically with laser light. The ionization at the target surface results in electrons streaming away from the surface, dragging ions in their wake. The back reaction resulting from ion blow-off compresses the target and the aim of inertial confinement is to achieve compression around 1000 times

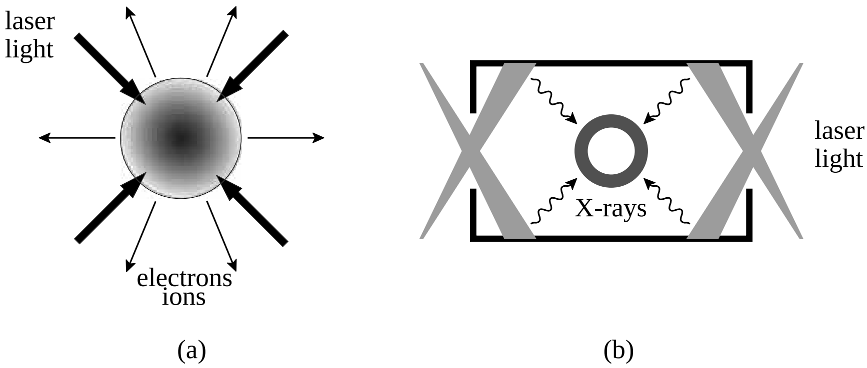

Fig. 1.3. Direct drive (a) and indirect drive (hohlraum) (b) irradiation of targets by intense laser light.

liquid density with minimal heating of the target until the final phase when the compressed fuel is heated to thermonuclear reaction temperatures. An alternative to the *direct drive* approach illustrated in Fig. 1.3(a) is shown in Fig. 1.3(b) in which the target is surrounded by a *hohlraum*. Light enters the hohlraum and produces X-rays which in turn provide target compression and *indirect drive* implosion.

1.3 Plasmas in space

Thermonuclear burn in stars is the source of plasmas in space. From stellar cores where thermonuclear fusion takes place, keV photons propagate outwards towards the surface, undergoing energy degradation through radiation–matter interactions on the way. In the case of the Sun the surface is a black body radiator with a temperature of 5800 K. Photons propagate outwards through the radiation zone across which the temperature drops from about 10^7 K in the core to around 5×10^5 K at the boundary with the convection zone. This boundary is marked by a drop in temperature so steep that radiative transfer becomes unstable and is supplanted as the dominant mode of energy transport by the onset of convection.

Just above the convection zone lies the photosphere, the visible 'surface' of the Sun, in the sense that photons in the visible spectrum escape from the photosphere. UV and X-ray surfaces appear at greater heights. Within the photosphere the Sun's temperature falls to about 4300 K and then unexpectedly begins to rise, a transition that marks the boundary between photosphere and chromosphere. At the top of the chromosphere temperatures reach around 20 000 K and heating then surges dramatically to give temperatures of more than a million degrees in the corona.

The surface of the Sun is characterized by magnetic structures anchored in the photosphere. Not all magnetic field lines form closed loops; some do not close in the photosphere with the result that plasma flowing along such field lines is not bound to the Sun. This outward flow of coronal plasma in regions of open magnetic field constitutes the solar wind. The interaction between this wind and the Earth's magnetic field is of great interest in the physics of the Sun–Earth plasma system. The Earth is surrounded by an enormous magnetic cavity known as the magnetosphere at which the solar wind is deflected by the geomagnetic field, with dramatic consequences for each. The outer boundary of the magnetosphere occurs at about $10R_E$, where R_E denotes the Earth's radius. The geomagnetic field is swept into space in the form of a huge cylinder many millions of kilometres in length, known as the magnetotail. Perhaps the most dramatic effect on the solar wind is the formation of a shock some $5R_E$ upstream of the magnetopause, known as the bow shock. We shall discuss a number of these effects later in the book by way of illustrating basic aspects of the physics of plasmas.

1.4 Plasma characteristics

We now introduce a number of concepts fundamental to the nature of any plasma whatever its origin. First we need to go a step beyond our statement in Section 1.1 and obtain a more formal identification of the plasma condition. Perhaps the most notable feature of a plasma is its ability to maintain a state of charge neutrality. The combination of low electron inertia and strong electrostatic field, which arises from even the slightest charge imbalance, results in a rapid flow of electrons to re-establish neutrality.

The first point to note concerns the nature of the electrostatic field. Although at first sight it might appear that the Coulomb force due to any given particle extends over the whole volume of the plasma, this is in fact not the case. Debye, in the context of electrolytic theory, was the first to point out that the field due to any charge imbalance is shielded so that its influence is effectively restricted to within a finite range. For example, we may suppose that an additional ion with charge Ze is introduced at a point P in an otherwise neutral plasma. The effect will be to attract electrons towards P and repel ions away from P so that the ion is surrounded by a neutralizing 'cloud'. Ignoring ion motion and assuming that the number density of the electron cloud n_c is given by the Boltzmann distribution, $n_c = n_e \exp(e\phi/k_B T_e)$, where T_e is the electron temperature, we solve Poisson's equation for the electrostatic potential $\phi(r)$ in the plasma.

Since $\phi(r) \to 0$ as $r \to \infty$, we may expand $\exp(e\phi/k_B T_e)$ and with $Zn_i = n_e$, Poisson's equation for large r and spherical symmetry about P becomes

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}\phi}{\mathrm{d}r}\right) = \frac{n_e e^2}{\varepsilon_0 k_B T_e}\phi = \frac{\phi}{\lambda_D^2} \tag{1.2}$$

say, where ε_0 is the vacuum permittivity. Now matching the solution of (1.2), $\phi \sim \exp(-r/\lambda_D)/r$, with the potential $\phi = Ze/4\pi\varepsilon_0 r$ as $r \to 0$ we see that

$$\phi(r) = \frac{Ze}{4\pi\varepsilon_0 r}\exp(-r/\lambda_D) \tag{1.3}$$

where

$$\lambda_D = \left(\frac{\varepsilon_0 k_B T_e}{n_e e^2}\right)^{1/2} \simeq 7.43 \times 10^3 \left(\frac{T_e(\text{eV})}{n_e}\right)^{1/2} \text{ m} \tag{1.4}$$

is called the *Debye shielding length*. Beyond a *Debye sphere*, a sphere of radius λ_D, centred at P, the plasma remains effectively neutral. By the same argument λ_D is also a measure of the penetration depth of external electrostatic fields, i.e. of the thickness of the boundary sheath over which charge neutrality may not be maintained.

The plausibility of the argument used to establish (1.3) requires that a large number of electrons be present within the Debye sphere, i.e. $n_e\lambda_D^3 \gg 1$. The inverse of this number is proportional to the ratio of potential energy to kinetic energy in the plasma and may be expressed as

$$g = \frac{e^2}{\varepsilon_0 k_B T_e \lambda_D} = \frac{1}{n_e \lambda_D^3} \ll 1 \tag{1.5}$$

Since g plays a key role in the development of formal plasma theory it is known as the *plasma parameter*. Broadly speaking, the more particles there are in the Debye sphere the less likely it is that there will be a significant resultant force on any given particle due to ‘collisions’. It is, therefore, a measure of the dominance of collective interactions over collisions.

The most fundamental of these collective interactions are the *plasma oscillations* set up in response to a charge imbalance. The strong electrostatic fields which drive the electrons to re-establish neutrality cause oscillations about the equilibrium position at a characteristic frequency, the *plasma frequency* ω_p. Since the imbalance occurs over a distance λ_D and the electron thermal speed V_e is typically $(k_B T_e/m_e)^{1/2}$ we may express the electron plasma frequency ω_{pe} by

$$\omega_{pe} = \frac{(k_B T_e/m_e)^{1/2}}{\lambda_D} = \left(\frac{n_e e^2}{m_e \varepsilon_0}\right)^{1/2} \tag{1.6}$$

which reduces to $\omega_{pe} \simeq 56.4 n_e^{1/2}\,\mathrm{s}^{-1}$. Note that any applied fields with frequencies less than the electron plasma frequency are prevented from penetrating the plasma by the more rapid electron response which neutralizes the field. Thus a plasma is not transparent to electromagnetic radiation of frequency $\omega < \omega_{pe}$. The corresponding frequency for ions, the *ion plasma frequency* ω_{pi}, is defined by

$$\omega_{pi} = \left(\frac{n_i (Ze)^2}{m_i \varepsilon_0}\right)^{1/2} \simeq 1.32 Z \left(\frac{n_i}{A}\right)^{1/2} \tag{1.7}$$

where Z denotes the charge state and A the atomic number.

1.4.1 Collisions and the plasma parameter

We have seen that the effective range of an electric field, and hence of a collision, is the Debye length λ_D. Thus any particle interacts at any instant with the large number of particles in its Debye sphere. Plasma collisions are therefore *many-body interactions* and since $g \ll 1$ collisions are predominantly weak, in sharp contrast with the strong, binary collisions that characterize a neutral gas. In gas kinetics a collision frequency ν_c is defined by $\nu_c = n V_{th} \sigma(\pi/2)$ where $\sigma(\pi/2)$ denotes the cross-section for scattering through $\pi/2$ and V_{th} is a thermal velocity. Such a deflection in a plasma would occur for particles 1 and 2 interacting over a distance b_0 for which $e_1 e_2 / 4\pi\varepsilon_0 b_0 \sim k_B T$ so that $\nu_c = (n V_{th} \pi b_0^2)$. However, the cumulative effect of the much more frequent weak interactions acts to increase this by a factor $\sim 8 \ln(\lambda_D / b_0) \approx 8 \ln(4\pi n \lambda_D^3)$. For electron collisions with ions of charge Ze it follows that the electron–ion collision time $\tau_{ei} \equiv \nu_{ei}^{-1}$ is given by

$$\tau_{ei} = \frac{2\pi \varepsilon_0^2 m_e^{1/2} (k_B T_e)^{3/2}}{Z^2 n_i e^4 \ln \Lambda} \tag{1.8}$$

where $\ln \Lambda = \ln 4\pi n \lambda_D^3$ is known as the *Coulomb logarithm*. For singly charged ions the electron–ion collision time is

$$\tau_{ei} = 3.44 \times 10^{11} \frac{T_e^{3/2}\,(\mathrm{eV})}{n_i \ln \Lambda}\,\mathrm{s}$$

in which we have replaced the factor 2π in (1.8) with the value found from a correct treatment of plasma transport in Chapter 12. The Coulomb logarithm is

$$\ln \Lambda = 6.6 - \frac{1}{2} \ln\left(\frac{n}{10^{20}}\right) + \frac{3}{2} \ln T_e\,(\mathrm{eV})$$

The *electron mean free path* $\lambda_e = V_e \tau_{ei}$ is

$$\lambda_e = 1.44 \times 10^{17} \frac{T_e^2 (\mathrm{eV})}{n_i \ln \Lambda}$$

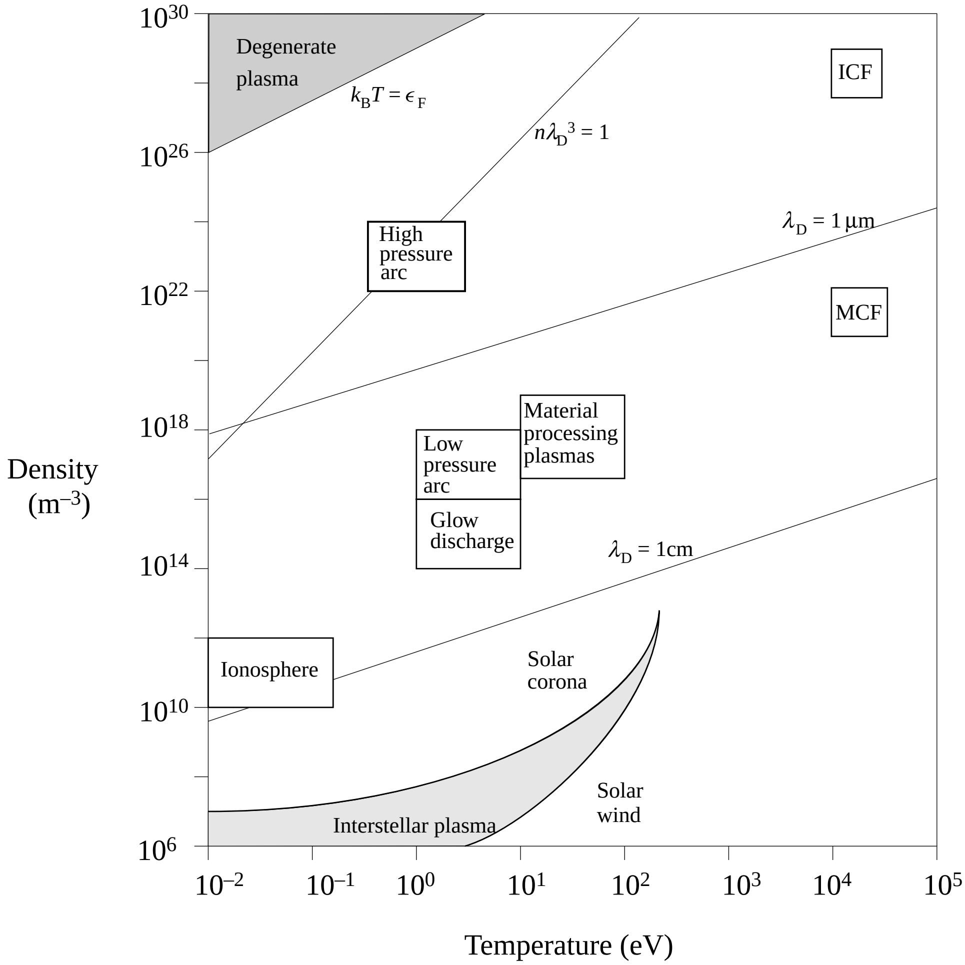

Fig. 1.4. Landmarks in the plasma universe.

Table 1.1 lists approximate values of various plasma parameters along with typical values of the magnetic field associated with each for a range of plasmas across the plasma universe. These and other representative plasmas are included in the diagram of parameter space in Fig. 1.4 which includes the parameter lines $\lambda_D = 1\,\mu m$, $1\,cm$ and $n\lambda_D^3 = 1$ together with the line marking the boundary at which plasmas become degenerate $k_BT = \epsilon_F$, where ϵ_F denotes the Fermi energy.

Table 1.1. *Approximate values of parameters across the plasma universe.*

Plasma	n (m^{-3})	T (keV)	B (T)	ω_{pe} (s^{-1})	λ_D (m)	$n\lambda_D^3$	ν_{ei} (Hz)
Interstellar	10^6	10^{-5}	10^{-9}	$6 \cdot 10^4$	0.7	$3 \cdot 10^5$	$4 \cdot 10^8$
Solar wind (1 AU)	10^7	10^{-2}	10^{-8}	$2 \cdot 10^5$	7	$4 \cdot 10^9$	10^{-4}
Ionosphere	10^{12}	10^{-4}	10^{-5}	$6 \cdot 10^7$	$2 \cdot 10^{-3}$	10^4	10^4
Solar corona	10^{12}	0.1	10^{-3}	$6 \cdot 10^7$	0.07	$4 \cdot 10^8$	0.5
Arc discharge	10^{20}	10^{-3}	0.1	$6 \cdot 10^{11}$	$7 \cdot 10^{-7}$	40	10^{10}
Tokamak	10^{20}	10	10	$6 \cdot 10^{11}$	$7 \cdot 10^{-5}$	$3 \cdot 10^7$	$4 \cdot 10^4$
ICF	10^{28}	10	—	$6 \cdot 10^{15}$	$7 \cdot 10^{-9}$	$4 \cdot 10^3$	$4 \cdot 10^{11}$