

Cambridge University Press

978-0-521-45253-3 - Fractals, Scaling and Growth Far from Equilibrium

Paul Meakin

Table of Contents

[More information](#)

Contents

Preface *xiii***Chapter 1 Pattern Formation Far From Equilibrium 1**

1.1	Power Laws and Scaling	4
1.2	The Logistic Map	16
1.3	The Variety of Patterns in Nature	22
1.3.1	<i>Euclidean Patterns</i>	24
1.3.2	<i>Cellular Patterns</i>	27
1.3.3	<i>Spiral and Helix Patterns</i>	31
1.3.4	<i>Labyrinthine Patterns</i>	32
1.3.5	<i>Fluid Convection Patterns</i>	34
1.4	Moving-Boundary Processes	36
1.4.1	<i>Solidification</i>	37
1.4.2	<i>Growth from Solution</i>	39
1.4.3	<i>Solidification of Impure Materials</i>	42
1.4.4	<i>Viscous Fingering</i>	44
1.4.5	<i>Pattern Selection</i>	45
1.4.6	<i>Anisotropy and Growth Velocity</i>	46
1.4.7	<i>Laplacian Growth</i>	49
1.4.8	<i>Instabilities</i>	49
1.4.9	<i>Characteristic Lengths</i>	50
1.4.10	<i>Beyond Linear-Stability Analysis</i>	51
1.5	Solution of Interface Equations of Motion	52

1.5.1	<i>Numerical Solution of the Non-Local Equations</i>	52
1.5.2	<i>Local Models</i>	53
1.6	<i>Complex and Disorderly Patterns</i>	57
1.6.1	<i>Aggregates</i>	59
1.6.2	<i>Polymers</i>	60
1.7	<i>Scaling Symmetry</i>	61
1.8	<i>Notation</i>	62
1.9	<i>Monte Carlo Methods</i>	62
1.10	<i>Additional Information</i>	64

Chapter 2 Fractals and Scaling 65

2.1	<i>Self-Similar Fractals</i>	65
2.1.1	<i>Statistical Self-Similarity</i>	69
2.1.2	<i>Lacunarity</i>	70
2.1.3	<i>Determination of the Fractal Dimensionality</i>	74
2.1.4	<i>The Devil's Staircase</i>	81
2.2	<i>Simple Rules</i>	83
2.3	<i>Finite-Size Effects and Crossovers</i>	85
2.4	<i>Power Law Distributions</i>	100
2.5	<i>Scaling</i>	104
2.5.1	<i>Corrections to Scaling</i>	111
2.5.2	<i>Multiscaling</i>	112
2.6	<i>Fractal Trees and Inhomogeneous Fractals</i>	113
2.7	<i>Self-Affine Fractals</i>	119
2.7.1	<i>Generation of Self-Affine Surfaces</i>	124
2.7.2	<i>The Geometry and Growth of Rough Surfaces</i>	130
2.7.3	<i>Characterization of Self-Affine Rough Surfaces</i>	135
2.7.4	<i>Finite-Size Effects and Crossovers</i>	152
2.7.5	<i>Status</i>	153
2.7.6	<i>Long Range Persistence</i>	155
2.8	<i>Multifractals</i>	160
2.9	<i>Universality</i>	165
2.10	<i>Additional Information</i>	166

Chapter 3 Growth Models 168

3.1	<i>Cluster Growth and Cluster Surfaces</i>	169
3.2	<i>Lattice Animals</i>	172
3.3	<i>Random Walks</i>	173

Contents

ix

3.3.1	<i>Self-Avoiding Random Walks</i>	174
3.3.2	<i>Indefinitely Growing Walks</i>	176
3.3.3	<i>The Diffusion-Limited Growth Walk</i>	177
3.3.4	<i>Random Walks on Random Substrates</i>	181
3.3.5	<i>Active Random Walk Models</i>	182
3.4	<i>Cluster Growth Models</i>	183
3.4.1	<i>The Eden Model</i>	184
3.4.2	<i>Ballistic Aggregation</i>	187
3.4.3	<i>The Diffusion-Limited Aggregation Model</i>	189
3.4.4	<i>The Dielectric Breakdown Model</i>	193
3.4.5	<i>The Scaling Structure of DLA</i>	198
3.4.6	<i>Other Aspects of DLA</i>	210
3.4.7	<i>Diffusion-Limited Annihilation</i>	211
3.5	<i>Percolation and Invasion Percolation</i>	214
3.5.1	<i>Growth Models for Percolation</i>	229
3.5.2	<i>Invasion Percolation</i>	231
3.5.3	<i>Diffusion Fronts and the Effect of Gradients</i>	234
3.5.4	<i>Directed Percolation</i>	239
3.5.5	<i>The Screened Growth Model</i>	242
3.5.6	<i>Faceted Growth Models</i>	243
3.6	<i>Packing Models</i>	246
3.7	<i>Growth Models Related to DLA</i>	250
3.7.1	<i>Homogeneous Perturbations</i>	253
3.7.2	<i>Inhomogeneous Perturbations</i>	256
3.7.3	<i>Attractive Interaction Model</i>	269
3.7.4	<i>Growth on Fibers and Surfaces</i>	272
3.7.5	<i>Simplified DLA Models</i>	279
3.8	<i>Noise Reduction and Deterministic Models</i>	285
3.8.1	<i>Lattice Structure Effects</i>	291
3.9	<i>Models with Quenched Disorder</i>	295
3.9.1	<i>Growth in High-Dimensionality Spaces</i>	297
3.10	<i>Theoretical Methods</i>	299
3.10.1	<i>Mean Field Theories</i>	302
3.10.2	<i>Wedge Growth Theories</i>	306
3.10.3	<i>Real-Space Renormalization Theories</i>	316
3.10.4	<i>Other Approaches</i>	319
3.11	<i>Additional Information</i>	325

Chapter 4	Experimental Studies	326
4.1	DLA Processes	327
4.1.1	<i>Electrochemical Deposition</i>	328
4.1.2	<i>Fluid–Fluid Displacement Experiments</i>	342
4.1.3	<i>Thin Films and Interfaces</i>	348
4.1.4	<i>Dissolution, Melting and Erosion of Porous Media</i>	356
4.1.5	<i>Solidification and Crystallization</i>	360
4.1.6	<i>Dielectric Breakdown</i>	363
4.1.7	<i>Growth Probability Distributions</i>	364
4.2	Dense Branching Morphology	366
4.2.1	<i>Electrochemical Deposition</i>	369
4.2.2	<i>Thin Films</i>	375
4.2.3	<i>Fluid–Fluid Displacement</i>	377
4.2.4	<i>Spherulites</i>	380
4.3	Percolation	381
4.4	Invasion Percolation	384
4.5	Displacement in Complex Fluids	388
4.5.1	<i>Polymer Solutions</i>	389
4.5.2	<i>Colloidal Systems</i>	389
4.5.3	<i>Foams</i>	393
4.5.4	<i>Fractal Systems</i>	394
4.6	Other 2-Dimensional Patterns	397
4.7	Additional Information	400
Chapter 5	The Growth of Surfaces and Interfaces	401
5.1	The Structure and Growth of Rough Surfaces	404
5.1.1	<i>Basic Surface Growth Equations</i>	405
5.1.2	<i>Surface Diffusion</i>	408
5.1.3	<i>Universality Classes</i>	411
5.1.4	<i>Exponent Scaling Relationships</i>	415
5.1.5	<i>The Kuramoto–Sivashinsky Equation</i>	417
5.2	Simple Models	418
5.2.1	<i>Eden Growth Models</i>	419
5.2.2	<i>Ballistic Deposition Models</i>	420
5.2.3	<i>Solid-on-Solid Models</i>	425
5.2.4	<i>The Polynuclear Growth Model</i>	428
5.2.5	<i>Directed Polymers</i>	429
5.2.6	<i>Langevin Dynamics Simulations</i>	432

Contents

xi

5.2.7	<i>Directed Percolation</i>	433
5.3	<i>Theoretically Motivated Models</i>	434
5.3.1	<i>Surface Growth with Weak Non-linearity</i>	434
5.3.2	<i>Correlated Noise</i>	439
5.3.3	<i>Non-Gaussian Noise</i>	445
5.3.4	<i>Growth on Rough Substrates</i>	449
5.4	<i>Models with Quenched Disorder</i>	450
5.4.1	<i>Models and Simulation Results</i>	452
5.4.2	<i>Universality Classes</i>	465
5.4.3	<i>Exponent Scaling Relationships</i>	469
5.5	<i>Experiments</i>	475
5.5.1	<i>Fluid–Fluid Displacement Experiments</i>	476
5.5.2	<i>The Growth of Cell Colonies</i>	483
5.5.3	<i>Phase Boundaries and Grain Boundaries</i>	484
5.5.4	<i>Deposition Experiments</i>	486
5.5.5	<i>Erosion Experiments</i>	510
5.5.6	<i>Electrochemical Deposition</i>	516
5.5.7	<i>Corrosion and Oxidation</i>	518
5.5.8	<i>Some General Comments</i>	519
5.6	<i>Thin Film Growth Models</i>	520
5.6.1	<i>The Effects of Surface Diffusion</i>	521
5.6.2	<i>Step Edge Dynamics</i>	546
5.6.3	<i>Anomalous Scaling</i>	547
5.6.4	<i>Porous and Amorphous Films</i>	549
5.6.5	<i>Anisotropic Surfaces</i>	551
5.6.6	<i>The Huygens Principle Model</i>	552
5.7	<i>Oblique Incidence and Shadowing Models</i>	553
5.7.1	<i>Oblique Incidence Ballistic Deposition Models</i>	553
5.7.2	<i>Ballistic Fans</i>	561
5.7.3	<i>Shadowing Models</i>	562
5.8	<i>Cluster Shapes and Faceted Growth</i>	569
5.9	<i>Additional Information</i>	573
	Appendix A Instabilities	574
A.1	<i>The Mullins–Sekerka Instability</i>	574
A.2	<i>The Saffman–Taylor Problem</i>	580

Cambridge University Press

978-0-521-45253-3 - Fractals, Scaling and Growth Far from Equilibrium

Paul Meakin

Table of Contents

[More information](#)

xii

Contents

	Appendix B Multifractals	585
B.1	Generation of Simple Multifractal Sets	586
B.2	Characterization of Multifractal Sets	591
B.3	Applications to Non-Equilibrium Growth	597
B.3.1	<i>Quenched and Annealed Averages</i>	605
B.3.2	<i>Mass Multifractals</i>	606
	References	608
	Index	663

Plate section between pp. 242 and 243*

*This plate section is available for download in color from
www.cambridge.org/9780521452533