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978-0-521-45253-3 - Fractals, Scaling and Growth Far from Equilibrium

Paul Meakin

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This book describes the progress that has been made towards the development of a comprehensive understanding of the formation of complex, disorderly patterns under far-from-equilibrium conditions.

The application of fractal geometry and scaling concepts to the quantitative description and understanding of structure formed under non-equilibrium conditions is described. Self-similar fractals, self-affine fractals, multifractals and scaling methods are discussed, with examples, to facilitate applications in the physical sciences. Computer simulations and experimental studies are emphasised, but the author also includes a discussion of theoretical advances in the subject. Much of the book deals with diffusion-limited growth processes and the evolution of rough surfaces, although a broad range of other applications is also included. The book concludes with an extensive reference list and guide to additional sources of information.

This book will be of interest to graduate students and researchers in physics, chemistry, materials science, engineering and the earth sciences, and especially those interested in applying the ideas of fractals and scaling to their work or those who have an interest in non-equilibrium phenomena.

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Contents

Preface xiii

Chapter 1	Pattern Formation Far From Equilibrium	1
1.1	Power Laws and Scaling	4
1.2	The Logistic Map	16
1.3	The Variety of Patterns in Nature	22
1.3.1	<i>Euclidean Patterns</i>	24
1.3.2	<i>Cellular Patterns</i>	27
1.3.3	<i>Spiral and Helix Patterns</i>	31
1.3.4	<i>Labyrinthine Patterns</i>	32
1.3.5	<i>Fluid Convection Patterns</i>	34
1.4	Moving-Boundary Processes	36
1.4.1	<i>Solidification</i>	37
1.4.2	<i>Growth from Solution</i>	39
1.4.3	<i>Solidification of Impure Materials</i>	42
1.4.4	<i>Viscous Fingering</i>	44
1.4.5	<i>Pattern Selection</i>	45
1.4.6	<i>Anisotropy and Growth Velocity</i>	46
1.4.7	<i>Laplacian Growth</i>	49
1.4.8	<i>Instabilities</i>	49
1.4.9	<i>Characteristic Lengths</i>	50
1.4.10	<i>Beyond Linear-Stability Analysis</i>	51
1.5	Solution of Interface Equations of Motion	52

1.5.1	<i>Numerical Solution of the Non-Local Equations</i>	52
1.5.2	<i>Local Models</i>	53
1.6	Complex and Disorderly Patterns	57
1.6.1	<i>Aggregates</i>	59
1.6.2	<i>Polymers</i>	60
1.7	Scaling Symmetry	61
1.8	Notation	62
1.9	Monte Carlo Methods	62
1.10	Additional Information	64
Chapter 2	Fractals and Scaling	65
2.1	Self-Similar Fractals	65
2.1.1	<i>Statistical Self-Similarity</i>	69
2.1.2	<i>Lacunarity</i>	70
2.1.3	<i>Determination of the Fractal Dimensionality</i>	74
2.1.4	<i>The Devil's Staircase</i>	81
2.2	Simple Rules	83
2.3	Finite-Size Effects and Crossovers	85
2.4	Power Law Distributions	100
2.5	Scaling	104
2.5.1	<i>Corrections to Scaling</i>	111
2.5.2	<i>Multiscaling</i>	112
2.6	Fractal Trees and Inhomogeneous Fractals	113
2.7	Self-Affine Fractals	119
2.7.1	<i>Generation of Self-Affine Surfaces</i>	124
2.7.2	<i>The Geometry and Growth of Rough Surfaces</i>	130
2.7.3	<i>Characterization of Self-Affine Rough Surfaces</i>	135
2.7.4	<i>Finite-Size Effects and Crossovers</i>	152
2.7.5	<i>Status</i>	153
2.7.6	<i>Long Range Persistence</i>	155
2.8	Multifractals	160
2.9	Universality	165
2.10	Additional Information	166
Chapter 3	Growth Models	168
3.1	Cluster Growth and Cluster Surfaces	169
3.2	Lattice Animals	172
3.3	Random Walks	173

3.3.1	<i>Self-Avoiding Random Walks</i>	174
3.3.2	<i>Indefinitely Growing Walks</i>	176
3.3.3	<i>The Diffusion-Limited Growth Walk</i>	177
3.3.4	<i>Random Walks on Random Substrates</i>	181
3.3.5	<i>Active Random Walk Models</i>	182
3.4	<i>Cluster Growth Models</i>	183
3.4.1	<i>The Eden Model</i>	184
3.4.2	<i>Ballistic Aggregation</i>	187
3.4.3	<i>The Diffusion-Limited Aggregation Model</i>	189
3.4.4	<i>The Dielectric Breakdown Model</i>	193
3.4.5	<i>The Scaling Structure of DLA</i>	198
3.4.6	<i>Other Aspects of DLA</i>	210
3.4.7	<i>Diffusion-Limited Annihilation</i>	211
3.5	<i>Percolation and Invasion Percolation</i>	214
3.5.1	<i>Growth Models for Percolation</i>	229
3.5.2	<i>Invasion Percolation</i>	231
3.5.3	<i>Diffusion Fronts and the Effect of Gradients</i>	234
3.5.4	<i>Directed Percolation</i>	239
3.5.5	<i>The Screened Growth Model</i>	242
3.5.6	<i>Faceted Growth Models</i>	243
3.6	<i>Packing Models</i>	246
3.7	<i>Growth Models Related to DLA</i>	250
3.7.1	<i>Homogeneous Perturbations</i>	253
3.7.2	<i>Inhomogeneous Perturbations</i>	256
3.7.3	<i>Attractive Interaction Model</i>	269
3.7.4	<i>Growth on Fibers and Surfaces</i>	272
3.7.5	<i>Simplified DLA Models</i>	279
3.8	<i>Noise Reduction and Deterministic Models</i>	285
3.8.1	<i>Lattice Structure Effects</i>	291
3.9	<i>Models with Quenched Disorder</i>	295
3.9.1	<i>Growth in High-Dimensionality Spaces</i>	297
3.10	<i>Theoretical Methods</i>	299
3.10.1	<i>Mean Field Theories</i>	302
3.10.2	<i>Wedge Growth Theories</i>	306
3.10.3	<i>Real-Space Renormalization Theories</i>	316
3.10.4	<i>Other Approaches</i>	319
3.11	<i>Additional Information</i>	325

Chapter 4	Experimental Studies	326
4.1	DLA Processes	327
4.1.1	<i>Electrochemical Deposition</i>	328
4.1.2	<i>Fluid–Fluid Displacement Experiments</i>	342
4.1.3	<i>Thin Films and Interfaces</i>	348
4.1.4	<i>Dissolution, Melting and Erosion of Porous Media</i>	356
4.1.5	<i>Solidification and Crystallization</i>	360
4.1.6	<i>Dielectric Breakdown</i>	363
4.1.7	<i>Growth Probability Distributions</i>	364
4.2	Dense Branching Morphology	366
4.2.1	<i>Electrochemical Deposition</i>	369
4.2.2	<i>Thin Films</i>	375
4.2.3	<i>Fluid–Fluid Displacement</i>	377
4.2.4	<i>Spherulites</i>	380
4.3	Percolation	381
4.4	Invasion Percolation	384
4.5	Displacement in Complex Fluids	388
4.5.1	<i>Polymer Solutions</i>	389
4.5.2	<i>Colloidal Systems</i>	389
4.5.3	<i>Foams</i>	393
4.5.4	<i>Fractal Systems</i>	394
4.6	Other 2-Dimensional Patterns	397
4.7	Additional Information	400
Chapter 5	The Growth of Surfaces and Interfaces	401
5.1	The Structure and Growth of Rough Surfaces	404
5.1.1	<i>Basic Surface Growth Equations</i>	405
5.1.2	<i>Surface Diffusion</i>	408
5.1.3	<i>Universality Classes</i>	411
5.1.4	<i>Exponent Scaling Relationships</i>	415
5.1.5	<i>The Kuramoto–Sivashinsky Equation</i>	417
5.2	Simple Models	418
5.2.1	<i>Eden Growth Models</i>	419
5.2.2	<i>Ballistic Deposition Models</i>	420
5.2.3	<i>Solid-on-Solid Models</i>	425
5.2.4	<i>The Polynuclear Growth Model</i>	428
5.2.5	<i>Directed Polymers</i>	429
5.2.6	<i>Langevin Dynamics Simulations</i>	432

5.2.7	<i>Directed Percolation</i>	433
5.3	Theoretically Motivated Models	434
5.3.1	<i>Surface Growth with Weak Non-linearity</i>	434
5.3.2	<i>Correlated Noise</i>	439
5.3.3	<i>Non-Gaussian Noise</i>	445
5.3.4	<i>Growth on Rough Substrates</i>	449
5.4	Models with Quenched Disorder	450
5.4.1	<i>Models and Simulation Results</i>	452
5.4.2	<i>Universality Classes</i>	465
5.4.3	<i>Exponent Scaling Relationships</i>	469
5.5	Experiments	475
5.5.1	<i>Fluid–Fluid Displacement Experiments</i>	476
5.5.2	<i>The Growth of Cell Colonies</i>	483
5.5.3	<i>Phase Boundaries and Grain Boundaries</i>	484
5.5.4	<i>Deposition Experiments</i>	486
5.5.5	<i>Erosion Experiments</i>	510
5.5.6	<i>Electrochemical Deposition</i>	516
5.5.7	<i>Corrosion and Oxidation</i>	518
5.5.8	<i>Some General Comments</i>	519
5.6	Thin Film Growth Models	520
5.6.1	<i>The Effects of Surface Diffusion</i>	521
5.6.2	<i>Step Edge Dynamics</i>	546
5.6.3	<i>Anomalous Scaling</i>	547
5.6.4	<i>Porous and Amorphous Films</i>	549
5.6.5	<i>Anisotropic Surfaces</i>	551
5.6.6	<i>The Huygens Principle Model</i>	552
5.7	Oblique Incidence and Shadowing Models	553
5.7.1	<i>Oblique Incidence Ballistic Deposition Models</i>	553
5.7.2	<i>Ballistic Fans</i>	561
5.7.3	<i>Shadowing Models</i>	562
5.8	Cluster Shapes and Faceted Growth	569
5.9	Additional Information	573
	Appendix A Instabilities	574
A.1	The Mullins–Sekerka Instability	574
A.2	The Saffman–Taylor Problem	580

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Frontmatter
[More information](#)

xii

Contents

	Appendix B	Multifractals	585
B.1	Generation of Simple Multifractal Sets		586
B.2	Characterization of Multifractal Sets		591
B.3	Applications to Non-Equilibrium Growth		597
B.3.1	<i>Quenched and Annealed Averages</i>		605
B.3.2	<i>Mass Multifractals</i>		606
	References		608
	Index		663

Plate section between pp. 242 and 243*

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Preface

The development of a full understanding of the universe around us, in terms of the basic properties of fundamental “particles” and their interactions, has long been a dream of the physicist. Mindful of the difficulties encountered when this approach is used to calculate the behavior of very simple systems, such as molecules containing just a few atoms, the problem of understanding the nature of much more complex systems, such as snowflakes, soot aggregates and rough surfaces produced by processes such as vapor deposition or erosion, might seem to be a daunting prospect. However, during the past one or two decades, substantial progress has been made, based on statistical physics concepts such as scaling and the independent development of fractal geometry, based on late 19th century and early 20th century mathematics, by Benoit Mandelbrot. To a large extent, this progress has been made by giving up the idea that an understanding of complex systems can be based on an ever more detailed knowledge of their microscopic components and focusing instead on the “universal” properties that all materials possess in common, irrespective of their atomic and molecular structure, and the manner in which properties on one length scale relate to those on other length scales. The connection between microscopic and macroscopic behavior is still important, and the theoretical justification for much of the work described in this book is based on models that contain microscopic components and interactions, at least on an abstract level. However, one of the objectives of this book is to illustrate that scaling symmetries can be used, in much the same way as other symmetries, to study a wide variety of systems and phenomena, without taking into account the underlying microscopic physics on a detailed level.

One of the main objectives of this book is to show how a surprisingly wide range of complex, disorderly systems can be quantitatively understood using simple statistical physics concepts and simple mathematical tools. This book contains many equations, but it should be accessible to anyone with a good undergraduate education in the physical sciences. My original idea was to write a single volume on the basics of fractals and scaling and applications in various areas of science and technology. It soon became apparent that I wanted to say more than could reasonably be contained in one book. Consequently this book concentrates on some of the more fundamental aspects of pattern formation, fractals and scaling. I am in the process of writing a second book, focusing on colloidal fractals and aggregation kinetics, and a third monograph on the applications of fractals and scaling in selected areas of science and technology.

My own interest in this area was first stimulated by the work of Thomas Witten and Leonard Sander, more than ten years ago, on diffusion-limited aggregation. The diffusion-limited aggregation model has become one of the most important paradigms for disorderly growth, far from equilibrium, and plays a central role in this book.

Much of the work on this book was carried out during a one year visit to the Center for Advanced Studies at the Norwegian Academy of Science and Letters. The remainder of the work was carried out in the Physics Department at the University of Oslo. I would like to thank Jens Feder and especially Torstein Jøssang for hospitality at the University of Oslo, and Torstein Jøssang for making my stay at the Center for Advanced Studies possible. I have also benefited considerably from stimulating interactions with a quite large number of graduate students and post-doctoral associates at the University of Oslo.

I would like to thank Fereydoon Family, Joachim Krug, Leonard Sander, Lorraine Siperko, Tamás Vicsek and Stephanie Wunder for making valuable comments on a draft of this book and for making suggestions that have led to substantial improvements. I am also grateful to many colleagues and collaborators who have contributed figures; they are acknowledged in the figure captions. Many of the figures have been provided by graduate students in the Cooperative Phenomena Group at the University of Oslo and illustrate various aspects of their own research.

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