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978-0-521-45243-4 - Families of Exponentials: The Method of Moments in Controllability
Problems for Distributed Parameter Systems

Sergei A. Avdonin and Sergei A. Ivanov

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