This book presents the newly developed theory of nonharmonic Fourier series and its applications to the control of distributed parameter systems. The authors extend the theory to include vector exponential series.

The first part of the book presents the modern theory of nonharmonic Fourier series based on the geometry of Hilbert spaces. This approach permits the successful development of the theory of scalar exponential families and vector exponential families. The development of this mathematical apparatus paves the way for the second part of the book, which extends and upgrades the method of moments – one of the most powerful tools in the flourishing theory of the control of distributed parameter systems. The authors present the Fourier method for evolutionary equations in Hilbert space and investigate the deep connections between controllability and properties of exponentials. They go on to discuss the controllability of systems described by parabolic and hyperbolic PDEs for internal, boundary, initial, and pointwise control. Finally, they consider a number of applications to control problems of connected strings systems.

FAMILIES OF EXPONENTIALS

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The Method of Moments in Controllability Problems for Distributed Parameter Systems

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Frontmatter
More information

Notation

M, card M	– number of elements of M if M is finite and ∞ if M is infinite	
$\varphi_{\mathfrak{H}}(\mathfrak{M}, \mathfrak{N})$	– angle between subspaces	18
$\rightharpoonup$	– weak convergence of elements of a Hilbert space	
codim Ξ	– dimension of orthogonal complement of a linear set Ξ or dimension of the space orthogonal to all elements of family Ξ	
$\langle\cdot,\cdot\rangle$	– scalar product in auxiliary Hilbert space $\mathfrak{N}$	
$\langle\!\langle\cdot\rangle\!\rangle$	– norm in an auxiliary Hilbert space $\mathfrak{N}$	
$\delta(\sigma)$	– Carleson constant	52
(A_2)	– Muckenhoupt condition	95
K_S	– a subspace $H^2_+(\mathfrak{N}) \ominus SH^2_+(\mathfrak{N}) = \{f \in H^2_+(\mathfrak{N}) \mid f \perp SH^2_+(\mathfrak{N})\}$	48
K_a	– a subspace K_s with $S = \exp(ika)$	48
l^2_r	– Hilbert space of sequences with the norm $\|c\|_r = \left[\sum_{n=1}^{\infty} \|c_n\|^2(\lambda_n + \alpha)^r\right]^{1/2}$	147
W_r	– Hilbert space of functions $\sum_{n=1}^{\infty} c_n\varphi_n$ with the norm $\|f\|_{W_r} := \|\{c_n\}\|_r$	147
$\mathscr{W}_{r+1}$	– Hilbert space $W_{r+1} \oplus W_r$	154
$W(\rho_n)$	– Hilbert space of series $\sum_{n=1}^{\infty} c_n\varphi_n$ with the norm $(\sum \|c_n\|^2\rho_n^2)^{1/2}$	163
φ_n	– eigenfunction of operator A	
λ_n	– eigenvalue of operator A or a point of spectrum of exponential family	

ω_n	– eigenfrequency of operator A	
κ_n	– multiplicity of eigenvalue λ_n of operator A	
$\mathscr{E}$	– exponential family $\{e_n\}$, with elements $e_n = \exp(i\lambda_n)\eta_n$, where η_n belongs to an auxiliary Hilbert space	
$\tilde{\mathscr{E}}$	– exponential family $\{r_n e_n\}$	163, 170
$\mathscr{E}_0$	– exponential family $\{\rho_n e_n\}$	163, 170
x_λ	– simple fraction $x_\lambda(k) = \sqrt{\frac{\operatorname{Im}\lambda}{\pi}}\frac{1}{k-\bar{\lambda}}$	41
$\mathscr{X}_\Pi$	– family of vector simple fractions $\{x_\lambda \eta_\lambda\}_{\lambda\in\sigma}$	47
$\mathscr{X}'_\Pi$	– family biorthogonal to the family $\mathscr{X}_\Pi$	
$\mathscr{X}'_{\Theta,\Pi}$	– family biorthogonal to the family $P_\Theta \mathscr{X}_\Pi$	

Operations

Cl, Cl_H	– closure of a set in the norm of space H
Lin Ξ	– linear span of family Ξ
$\dot{+}$	– direct sum of linear sets
$\bigvee \Xi$	– closure of the linear span of family Ξ

Sets

$\mathbb{R}$	– set of real numbers	
$\mathbb{R}_+$	– set of positive numbers	
$\mathbb{Q}$	– set of rational numbers	
$\mathbb{Z}$	– set of integers	
$\mathbb{K}$	– set of nonzero integers	
$\mathbb{N}$	– set of positive integers	
$\mathbb{C}$	– set of complex numbers (complex plane)	
$\mathbb{C}^n$	– Cartesian product of n complex planes	
$\mathbb{C}_+$ ($\mathbb{C}_-$)	– open upper (lower) half-plane, i.e., set of complex numbers with positive (negative) imaginary part	
$\mathbb{D}$	– open unit disk in $\mathbb{C}$ with the center in origin	
$\mathbb{T}$	– unit circle in $\mathbb{C}$ with the center in origin	
$\mathscr{L}(X, Y)$	– set of linear bounded operators acting from space X into space Y	
$D(A)$	– domain of operator A	
D_Ξ	– domain of operator of the problem of moments with respect to family Ξ	34

R_Ξ	– image of operator of the problem of moments with respect to family Ξ	34
Ker A	– null space of operator A	

Operators

$P_{\mathfrak{M}}$	– orthoprojector on subspace $\mathfrak{N}$	
$\mathscr{P}_{\mathfrak{M}}^{\parallel \mathfrak{N}}$	– skew projector on subspace $\mathfrak{M}$ parallel to subspace $\mathfrak{N}$	20
I	– identical operator	
$\mathscr{J}_\Xi$	– operator of the problem of moments with respect to family Ξ	
P_S	– projector onto subspace K_S	48
$P_\pm$	– projectors onto subspace $H^2_\pm$	42

Conditions on countable sets of the upper half-plane

(B)	– Blaschke condition	43
(R)	– rareness condition	52
(C)	– Carleson condition	52
(CN)	– Carleson–Newman condition	52

Property of families of elements or subspaces of a Hilbert space

$\Xi \in (LB)$	– family Ξ forms a Riesz basis in closure of its linear span	26, 30
$\Xi \in (UM)$	– family Ξ of elements is *-uniformly minimal or family Ξ of subspaces is uniformly minimal	26, 30
$\Xi \in (M)$	– family Ξ is minimal	25, 30
$\Xi \in (W)$	– family Ξ is W-linear independent	24

Types of controllability

B controllability	162, 169
E controllability	162, 169
UM controllability	162, 169
M controllability	162, 169
W controllability	162, 169

Cancellations

$\mathscr{L}$-basis	– Riesz basis in closure of linear span	26, 30
BP	– Blaschke product	43
BPP	– Blaschke–Potapov product	44
DPS	– distributed parameter system	
ESF	– entire singular (operator) function	44
STF	– sine-type function	61
GF	– generating function	80, 101, 113

Signs

$A := B$ or $B =: A$ – object A is equal to object B by definition

$f(x) \asymp g(x),\ x \in X$ – this relation means that there are positive constants c and C such that for all $x \in X$ the inequalities $cg(x) \le f(x) \le Cg(x)$ are valid

$f(x) \prec g(x)$ or $g(x) \succ f(x),\ x \in X$ – this means that there exists a positive constant C such that for all $x \in X$ the inequality $f(x) \le Cg(x)$ is valid