The starting point of this book is Sperner's theorem, which answers the question: What is the maximum possible size of a family of pairwise unrelated (with respect to inclusion) subsets of a finite set? This theorem stimulated the development of a fast growing theory dealing with extremal problems on finite sets and, more generally, on finite partially ordered sets.

This book presents Sperner theory from a unified point of view, bringing combinatorial techniques together with methods from programming (e.g. flow theory and polyhedral combinatorics), from linear algebra (e.g. Jordan decompositions, Lie-algebra representations, and eigenvalue methods), from probability theory (e.g. limit theorems), and from enumerative combinatorics (e.g. Möbius inversion).

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PREFACE

Sperner theory is a very lively area of combinatorics and combinatorial optimization. Though nobody knows the exact age of it, we may assume that it was born in the end of the 1920s and that at least F.S. Macaulay and E. Sperner belong to the set of parents. Macaulay's contribution is discussed in the chapter on Macaulay posets. Sperner's theorem is the starting point of this book. It answers the following question: Given a family of pairwise unrelated (with respect to inclusion) subsets of a finite set, what is the maximum size of this family? In the 1930s the set of parents was enlarged by P. Erdős, C. Ko, and R. Rado, who studied intersection conditions, and in the beginning of the 1950s, R.P. Dilworth entered this set by proving his famous min–max theorem on partially ordered sets. We should not forget the grandfather. At the end of the last century R. Dedekind failed to find an explicit formula for the number of monotone Boolean functions, and this difficult problem was later attacked by several authors. It is impossible to mention all the offspring and descendants of Sperner theory. Many of them can be found in the references. The list of names there shows that many well-known mathematicians of this century have contributed to the development of the theory.

Several aspects make Sperner theory so interesting. The problems often may be clearly and simply formulated. For the solutions one indeed needs creativity, and the techniques are frequently surprising. Concerning applications and methods, there are several unexpected relations to other branches of mathematics, such as programming, algebra, probability theory, number theory, and geometry. Thus, studying Sperner theory means learning many important techniques in discrete mathematics and combinatorial optimization on a particular theme.

Ten years ago H.-D.O.F. Gronau and I wrote the monograph "Sperner theory in partially ordered sets" [161]. This work was followed by the books of B. Bollobás [73], I.A. Anderson [32], and C. Berge [50] that also discuss aspects of Sperner theory. Since I saw that a revision of [161] would be insufficient, I decided to write a new book. It has been my main goal to present a unifying theory that covers many seemingly distant and separate results, including as far as possible all important and famous theorems related to the subject. To meet this goal, I took a completely new approach toward selection, organization, and presentation of the material. Of course I also tried my best in order to give an updated presentation reflecting the modern development of the last ten years. More attention is paid to algorithmic aspects and much more space is devoted to the study of the Boolean lattice.

Up to a small epsilon everything is given with complete proofs (matched to the "theory"), so that the reader need not consult the original papers. The book is self-contained, and the reader needs only basic knowledge of mathematics. I hope that enthusiasm for the subject increases exponentially with the number of pages read.

Recognizing that it is impossible to refer to all of the large number of papers related to Sperner theory, I have put the emphasis on the main results and methods of the theory and not on a complete survey of all related results. Consequently, many topics are presented only in examples, and I apologize for omitting several interesting results.

I also intended to keep intersections with existing and forthcoming books small. For instance, I wrote nothing on dimension theory (W.T. Trotter [452]) and design theory (e.g., T. Beth, D. Jungnickel, H. Lenz [53]). The probabilistic methods are not discussed extensively because they are covered in the book by N. Alon and J.H. Spencer [29]. Some further algebraic methods will be contained in the forthcoming book by P. Frankl and L. Babai [35]; and L.H. Harper and J.D. Chavez [261] are building up a theory for discrete isoperimetric problems. Since there exist excellent books of problems as well as books containing problems with solutions like those of G.P. Gavrilov and A.A. Sapozhenko [220] and L. Lovász [354] and I.A. Anderson [32], I did not include exercises. However, several theorems, lemmata, propositions, corollaries, and claims in this book are suitable for exercises (which thus have complete solutions). Some *open problems* are mentioned in the text, and the corresponding page numbers can be found using the index.

The "easiest" examples for an author are those the author has studied. So my own list of references may seem inappropriately long. I included also some new results that will not be published elsewhere. Thus I hope the book might shed new light on some topics also for people who already do active research in the field.

I wish to thank Larry Harper, who initiated this project and who contributed with stimulating discussions. Many other mathematicians gave hints and indications to new results, and I gratefully acknowledge their contributions. Beyond that, I wish to express my gratitude to all my teachers, direct and indirect. In particular I would like to thank Christian Bey, Frank Ihlenburg, and Uwe Leck, who read the whole or at least parts of the manuscript very thoroughly. They found several unpleasant typographical and substantive errors, and I hope that their number is minimized now.

I am indebted to all involved in the production of this book. Cynthia Benn and Rena Wells were of great assistance in improving the style in many ways.

My deep thanks go to my parents Helga and Wolfgang Engel for (among many other things) introducing me early into mathematics as well as encouraging and supporting my occupation with it.

Above all I am grateful to my own family for opening widely happy new pages of my life, for their continuous encouragement and understanding. I leave it as an exercise to discover the names of my wife and my children in the book.