This book studies existence and necessary conditions, such as Pontryagin's maximum principle for optimal control problems described by ordinary and partial differential equations. These necessary conditions are obtained from Kuhn–Tucker theorems for nonlinear programming problems in infinite dimensional spaces.

The optimal control problems include control constraints, state constraints, and target conditions. Evolution partial differential equations are studied using semigroup theory, abstract differential equations in linear spaces, integral equations, and interpolation theory. Existence of optimal controls is established for arbitrary control sets by means of a general theory of relaxed controls.

Applications include nonlinear systems described by partial differential equations of hyperbolic and parabolic type; the latter case deals with pointwise constraints on the solution and the gradient. The book also includes results on convergence of suboptimal controls.

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To Natalia

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FOREWORD

An initial value or initial-boundary value problem for an evolution partial differential equation (an equation whose solutions depend on time) can usually be written as an abstract differential equation

$$y'(t) = f(t, y(t)) \tag{1}$$

in a suitable function space, the function f describing the action of the equation on the space variables, with boundary conditions (if any) included in the definition of the space or of the domain of f. The similarity of (1) with a true ordinary differential equation is only formal (f may not be everywhere defined, bounded or continuous) but gives heuristic insight into the problem, suggests ways to extend results from ordinary to partial differential equations and stresses unification, discovery of common threads and economy of thought. The "abstract" approach is not the best in all situations (for instance, many controllability results depend on properties of partial differential equations lost, or difficult to reformulate, in the translation to (1)), but it applies very well to optimization problems, where it is expedient to obtain general statements such as Pontryagin's maximum principle and then elucidate what the principle says for equations in various classes. Many of the techniques are (modulo some fine tuning) oblivious to the type of equation and are at least formally similar to classical procedures for systems of ordinary differential equations.

This work is on the Pontryagin's maximum principle for equations of the form (1), on its applications to diverse control systems described by partial differential equations, including control and state constraints and target conditions, and on other related questions such as existence and relaxation of controls. It is understood for use by nonspecialists, and with this in view incorporates blocks of auxiliary material (Sections **2.0**, **5.0** and **12.0** and portions of other chapters).

Those familiar with Pontryagin's maximum principle know that the key to its meaning lies not so much in its proof but in understanding what it says or does not say as applied to a particular optimal control problem, and finite dimensional systems are perhaps the best area where one can gain insight without much overhead. This motivates Part I (the first four chapters), which give a cursory introduction to some control problems for ordinary differential equations and a large number of examples, all classical in the literature. Part of the material (such as the nonlinear programming theory in Chapters 2 and 3) is also used later.

Part II (Chapters 5 to 11) is on infinite dimensional control systems, with Chapter 9 on linear systems. Linear problems are amenable to separation techniques more elementary than nonlinear programming, and there are linear theorems that do not yet have full nonlinear counterparts.

Part III (Chapters 12 to 14) is on relaxed controls; these appear when one tries to insure existence of optimal controls, something not always attainable with the "original" controls with which the problem is outfitted.

There are many obvious shortcuts through this book. To mention one, the fastest way to the infinite dimensional maximum principle begins with **7.1** and **7.2** on the general nonlinear programming problem in Banach spaces and then proceeds to the maximum principle with state constraints in Chapter 10, with assistance of various sections in Chapters 5 and 6; for the parabolic problems in Chapter 11, some of the material in Chapters 7 and 8 is needed.

All through, "Examples" are results either informally proved or left to the reader as exercises.

The references have no pretension of completeness. They only include works that deal with control problems through the abstract evolution equation (1) *and* are directly related with the results in this book, in particular with the maximum principle. When appropriate, we include papers that arrive at similar results by other methods.

We have also attempted to include a modicum of references to subjects not treated in any detail in this work (for instance, controllability, stabilization and the Hamilton-Jacobi approach to optimality); here, the words "... and other papers" invite the reader to perform further search. When possible, we have deferred to the extensive references in several recent books in control theory.

Acknowledgments

In 1986 I was invited by the Ministry of Education and Justice of Argentina to participate in the 1st. National School of Applied Mathematics, held that year in Potrero de Los Funes. I taught a course on finite dimensional control theory to an audience of graduate students from various Argentine universities. In 1987, a second National School of Applied Mathematics was held in Santa Fe de la Vera Cruz. The audience consisted of some of my 1986 students and some new ones, and the lectures were on infinite dimensional control theory, corresponding to various subjects in Parts II and III of this book. Notes were written by students on both occasions and were the initial impulse for the project. I am happy to acknowledge here my debt to the many individuals in the Secretary of Science and Technology and the National Research Council for this opportunity, as well as to the local organizers.

The second, and most important lucky break came with Halina Frankowska's visit to UCLA in 1987, during which the nonlinear programming theory underlying

most of this work came into being, as well as the strategy to approach problems with state constraints.

Later, S. S. Sritharan provided the impetus, the motivation and much of the technical means for application of the nonlinear programming theory to problems in fluid mechanics. Gieri Simonett read large parts of the manuscript and is responsible for many improvements. Finally, Tomáš Roubíček contributed substantially to Part III and Wolfgang Rueß clarified various functional analytic constructs. It goes without saying that none of the above authors is in any way responsible for errors, or misinterpretations of their results or suggestions.

During my scientific career, I had the good fortune to be able to attend many workshops on state-of-the art control theory. These included several Oberwolfach meetings and most of the Vorau conferences. The influence of the latter in the development of infinite dimensional control theory can hardly be overestimated; various concepts fundamental in this work were presented in Vorau conferences and published in their Proceedings.