

Cambridge University Press

0521450543 - The Geometry of Total Curvature on Complete Open Surfaces

Katsuhiko Shiohama, Takashi Shioya and Minoru Tanaka

Table of Contents

[More information](#)


---

## Contents

---

<i>Preface</i>	<i>page vii</i>
<b>1 Riemannian geometry</b>	<b>1</b>
1.1 The Riemannian metric	1
1.2 Geodesics	3
1.3 The Riemannian curvature tensor	8
1.4 The second fundamental form	14
1.5 The second variation formula and Jacobi fields	17
1.6 Index form	24
1.7 Complete Riemannian manifolds	28
1.8 The short-cut principle	31
1.9 The Gauss–Bonnet theorem	34
<b>2 The classical results of Cohn-Vossen and Huber</b>	<b>41</b>
2.1 The total curvature of complete open surfaces	41
2.2 The classical theorems of Cohn-Vossen and Huber	46
2.3 Special properties of geodesics on Riemannian planes	55
<b>3 The ideal boundary</b>	<b>75</b>
3.1 The curvature at infinity	75
3.2 Parallelism and pseudo-distance between curves	78
3.3 Riemannian half-cylinders and their universal coverings	91
3.4 The ideal boundary and its topological structure	94
3.5 The structure of the Tits metric $d_\infty$	100
3.6 Triangle comparison	105
3.7 Convergence to the limit cone	111
3.8 The behavior of Busemann functions	123

Cambridge University Press

0521450543 - The Geometry of Total Curvature on Complete Open Surfaces

Katsuhiro Shiohama, Takashi Shioya and Minoru Tanaka

Table of Contents

[More information](#)

vi

*Contents*

<b>4</b>	<b>The cut loci of complete open surfaces</b>	133
4.1	Preliminaries	133
4.2	The topological structure of a cut locus	140
4.3	Absolute continuity of the distance function of the cut locus	149
4.4	The structure of geodesic circles	156
<b>5</b>	<b>Isoperimetric inequalities</b>	165
5.1	The structures of $S(\mathcal{C}, t)$ and the cut locus of $\mathcal{C}$	165
5.2	The case where $M$ is finitely connected	169
5.3	The case where $M$ is infinitely connected	175
<b>6</b>	<b>Mass of rays</b>	187
6.1	Preliminaries; the mass of rays emanating from a fixed point	187
6.2	Asymptotic behavior of the mass of rays	195
<b>7</b>	<b>The poles and cut loci of a surface of revolution</b>	207
7.1	Properties of geodesics	207
7.2	Jacobi fields	218
7.3	The cut loci of a von Mangoldt surface	230
<b>8</b>	<b>The behavior of geodesics</b>	243
8.1	The shape of plane curves	243
8.2	Main theorems and examples	248
8.3	The semi-regularity of geodesics	252
8.4	Almost-regularity of geodesics; estimate of index	262
8.5	The rotation numbers of proper complete geodesics	266
8.6	The existence of complete geodesics arbitrarily close to infinity	270
	<i>References</i>	275
	<i>Index</i>	281