CAMBRIDGE STUDIES IN
ADVANCED MATHEMATICS 11

Local representation theory

Local representation theory

Modular representations as an introduction to the local representation theory of finite groups

J. L. ALPERIN

CAMBRIDGE
UNIVERSITY PRESS

Published by the Press Syndicate of the University of Cambridge
The Pitt Building, Trumpington Street, Cambridge CB2 1RP
40 West 20th Street, New York, NY 10011-4211, USA
10 Stamford Road, Oakleigh, Melbourne 3166, Australia

First published 1986
First paperback edition 1993

Library of Congress Cataloging-in-Publication Data available.

A catalogue record for this book is available from the British Library.

ISBN 0-521-44926-X paperback

Transferred to digital printing 2004

TO
GREGORY AND COURTNEY

Cambridge University Press
052144926X - Local Representation Theory: Modular Representations as an Introduction to the Local Representation Theory of Finite Groups
J. L. Alperin
Frontmatter
More information

Contents

Preface

The representation theory of finite groups is at the same time an old and well-developed subject and a quickly changing and dynamic one. This may be daunting to many readers but we propose a remedy: in this short text, despite the great breadth of the topic, we shall take the reader to one of the high points of the subject. Hence, we shall go very far very quickly. This emphasis on speed and height suggests an analogy with climbing a mountain and doing it in quite short order. Carrying this analogy further, it means that we must not burden ourselves down with lots of heavy equipment. This means, on the one hand, that we can move quickly up the mountain but that, on the other hand, there will be times where we will not have the best tools available. Specifically, in our case, we are going to limit ourselves to a single set of methods, even though there are several, no one of which is sufficient to achieve all the results, even the most important ones.

Local representation theory is the part of the subject that involves and relates representations in characteristic zero and in characteristic p, p a prime. This is analogous with many ideas in number theory. Local representation theory is also local in a second sense: the p-local subgroups play a central role. These subgroups, the normalizers of the non-identity p-subgroups, are critical here (as they are all over group theory) and arise in all the basic theorems.

Our plan is to study only kG-modules, where G is a finite group and k is an algebraically closed field of characteristic p. We shall not go into the characteristic zero consequences or the applications to group structure. In this way we can go very deeply in little time. After two introductory chapters, we shall prove the basic results of Green and use them, following along the lines of our work with Burry, to prove the fundamental results. In particular, we shall use the theorem of Burry–Carlson–Puig to establish

Cambridge University Press
052144926X - Local Representation Theory: Modular Representations as an Introduction to the Local Representation Theory of Finite Groups
J. L. Alperin
Frontmatter
More information

Brauer's First Main Theorem and then prove the module form of Brauer's Second Main Theorem due to Nagao. We shall then treat Feit's results connecting maps and the Green correspondence with a new module-theoretic argument. We shall conclude with the Brauer–Dade cyclic theory proceeding mainly along the lines of the Green–Peacock approach, except that the last few sections are new.

This is a text, so the reader will not find the completeness or best results that he would expect in a treatise, nor will he find any historical account. As a text this book is suitable for a one-semester course, with some deletions, or a year course with the addition of material on the relations between representation theory in characteristic p and zero and on the applications. The exercises have been carefully selected and a few are used later in the text.

We hope we can launch the reader into this broad and exciting subject. All of representation theory is divided into three parts. First, there is the general theory, second the representation theory of the most important groups, like the Lie type groups, and third there are connections with many other areas in that these other fields are used in studying representations and there are applications to these areas. These include structure of groups, number theory, algebras, homological algebra, combinatorics, orders, algebraic geometry and algebraic groups. We hope we have given an introduction which will stimulate the reader to explore all these wonderful ideas.