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PLASMA PHYSICS

An introduction to the theory of astrophysical, geophysical, and laboratory plasmas

PETER A. STURROCK

Center for Space Science and Astrophysics, Stanford University
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Preface

This book is based on a series of lectures that has been given at Stanford University, for longer than I care to remember, to graduate students from several departments: Aeronautics and Astronautics, Applied Physics, Electrical Engineering, Mechanical Engineering, and Physics. The course has also formed part of the Astronomy Course Program and of the Space Science Program.

The course has changed over the years, beginning as a three-quarter sequence emphasizing laboratory and geophysical plasmas, and evolving into a two-quarter sequence emphasizing solar and other astrophysical applications. Selected material has also been offered as a one-quarter course. The course has been much improved by input from many students (in fact, the first set of lecture notes was produced by students in the class) and from a sequence of dedicated teaching assistants, notably, in recent years, Dr Anton Bergmann, Ms Lisa Porter and Dr Yuri Taranenko.

For invaluable assistance in the preparation of this text, I am indebted to Mrs Louise Meyers-Norney, who entered the text, to Dr James and Mrs Maria Klimchuk, who entered the equations, and to Dr Taeil Bai and Mr David Faust, who helped prepare the figures. Thanks are due also to Dr George Field, Dr Robert Helliwell, Dr Eric Priest and Dr Gerard Van Hoven, who kindly reviewed some of the chapters, and to Dr Simon Mitton and Ms Fiona Thomson of Cambridge University Press for their generous support.

Some of the problems were developed as part of this course, but some were taken from other books. I regret that I did not keep records of the sources of all problems, and it is therefore inevitable that some problems are reproduced in the following pages without proper acknowledgement. For this delinquency I offer my apologies to the original authors. I would greatly appreciate being informed of these errors of omission – and also, of course, of errors of commission.
Preface

Finally, it is a pleasure to acknowledge my debt to my good friend Dr William Thompson, who first introduced me to plasma physics, and to thank Dr Marshall Rosenbluth, who has kindly permitted me to quote not only from his physics but also from his wit.