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Edited by Cynthia Hog-Angeloni, Wolfgang Metzler and Allan J. Sieradski

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# Two-Dimensional Homotopy and Combinatorial Group Theory

Edited by

Cynthia Hog-Angeloni  
*University of Frankfurt*

Wolfgang Metzler  
*University of Frankfurt*

and

Allan J. Sieradski  
*University of Oregon*



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## Editors' Preface

It is well known that techniques developed for manifolds of higher dimensions don't suffice to treat open problems in dimensions three and four. The latter are inextricably tied to questions about the (simple-) homotopy type of 2-skeleta or -spines of these low-dimensional manifolds and, hence, to presentations of groups.

Basic work on two-dimensional homotopy dates back to K. Reidemeister and J.H.C. Whitehead. For instance, Whitehead gave an algebraic description of the homotopy type of 2-complexes. But, until the early 70's, one didn't have examples of 2-complexes with different homotopy type but equal fundamental groups and Euler characteristic. Since then considerable advances have been made, yielding, in particular, remarkable partial results on famous open problems like Whitehead's question, whether subcomplexes of aspherical 2-complexes are always aspherical themselves. The authors of this book have contributed to this development.

Because of its relations to decision problems in combinatorial group theory, two-dimensional homotopy probably will never take the shape of a complete theory. However, the occurrence of certain notions (e.g., the Reidemeister-Peiffer identities of presentations) in different questions is far from being accidental. The time has come to collect the present knowledge in order to stimulate further research.

This book contains the elements of both a textbook and a research monograph, and, hence, addresses students as well as specialists. Parts of the book have already been used to substantiate courses with concrete geometric and/or algebraic material. A student reader should know already some (algebraic) topology and algebra. We start with two introductory chapters on low-dimensional complexes. These are followed by chapters on prominent techniques including their applications to manifolds. Concluding chapters treat the present status of three famous conjectures (Whitehead, Zeeman, Andrews-Curtis). The coherent organization of the book includes cross references as well as a common index and an ample bibliography. But the chapters can also be read independently; they range from an introduction to

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the specific topic(s) to a survey of latest results, with guidelines to the current literature. Particular emphasis is placed on covering open problems.

This book project was initiated by Wolfgang Metzler and presented to the majority of the authors at a workshop on Geometric Topology and Combinatorial Group Theory held in Luttach/Southern Tyrol (Italy) in August 1991. During this meeting, an approximate table of contents was developed.

The book demonstrates and documents mathematical cooperation. We mention, in particular, long-term activities of Micheal Dyer, Allan Sieradski and their former students, of Wolfgang Metzler and his former students and of Paul Latiolais' Fall Foliage Seminars, which have grown together in recent years. In addition to personal contact, modern electronic communication of (drafts of) sections or whole chapters was a basic ingredient in the production of manuscripts. This includes the illustrations that were based on sketches of the authors and drawn by Allan Sieradski using the Postscript drawing application Adobe Illustrator. The final layout was done by the editors.

All chapters were refereed twice, according to the usual textbook/journal standards, once by one or more member(s) of the team of authors, second by an external referee who had the option of remaining anonymous. As nobody chose to do so, we express our gratitude for their valuable service to: Juan Alonso (Stockholm), Stefan Bauer (Göttingen), Gerhard Burde (Frankfurt/Main), David Gillman (Los Angeles), Mauricio Gutierrez (Medford), Jens Harlander (Frankfurt/Main), Wolfgang Heil (Tallahassee), James Howie (Edinburgh), John Ratcliffe (Nashville), Nancy Waller (Portland) and Perrin Wright (Tallahassee). With their help, we have, in particular, tried to avoid mathematical and typographical errors. Our editorial efforts included eliminating conflicting use of terminology in different chapters; but we did not insist on standardized notations.

We invite all readers to communicate any remaining errors to us; we are also eager to learn about further progress on the mathematical substance of this book, which might be integrated into revisions of the text.

Last, but not least, we want to thank Roger Astley and David Tranah of Cambridge University Press for their continuous encouragement and advice throughout the project.

Cynthia Hog-Angeloni, Wolfgang Metzler, Allan J. Sieradski

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## Addresses of Authors

William A. Bogley  
Department of Mathematics  
Oregon State University  
Corvallis, OR 97331-4605 USA

Micheal N. Dyer  
Department of Mathematics  
University of Oregon  
Eugene, OR 97403 USA

Ian Hambleton  
Dept. of Mathematical Sciences  
McMaster University  
Hamilton, Ontario  
Canada L8S 4K1

Cynthia Hog-Angeloni  
Fachbereich Mathematik  
der Universität Frankfurt  
Postfach 11 19 32  
60054 Frankfurt am Main/Germany

Günther Huck  
Department of Mathematics  
Northern Arizona University  
Flagstaff, AZ 86011 USA

Matthias Kreck  
Fachbereich Mathematik  
der Universität Mainz  
Saarstrasse 21  
55122 Mainz/Germany

M. Paul Latiolais  
Dept. of Mathematical Sciences  
Portland State University  
Portland, OR 97207-0751 USA

Martin Lustig  
Institut für Mathematik  
der Ruhr-Universität  
Postfach 10 21 48  
44780 Bochum/Germany

Sergei Matveev  
Chelyabinsk State University  
129 Kashirin Brothers St.  
454136 Chelyabinsk/Russia

Wolfgang Metzler  
Fachbereich Mathematik  
der Universität Frankfurt  
Postfach 11 19 32  
60054 Frankfurt am Main/Germany

Steve J. Pride  
Department of Mathematics  
Glasgow University  
University Gardens  
Glasgow G12 8QW U.K.

Dale Rolfsen  
Department of Mathematics  
University of British Columbia  
Vancouver B.C. Canada V6T 1Y4

Stephan Rosebrock  
Fachbereich Mathematik  
der Universität Frankfurt  
Postfach 11 19 32  
60054 Frankfurt am Main/Germany

Allan J. Sieradski  
Department of Mathematics  
University of Oregon  
Eugene, OR 97403 USA