

Contents

<i>Preface</i>	<i>page</i>	ix
<i>Introduction to the student</i>		1
Part one: Simple Models in Mechanics		5
1 Newtonian mechanics		7
1.1 Mechanics before Newton		7
1.2 Kinematics and dynamics		10
1.3 Newton's laws		13
1.4 Gravity near the Earth		16
1.5 Units and dimensions		18
2 Kinematics on a line		21
2.1 Displacement and velocity		22
2.2 Acceleration		28
2.3 Derivatives as slopes		33
2.4 Differential equations and antiderivatives		37
3 Ropes and pulleys		41
3.1 Tension in the rope		41
3.2 Solving pulley problems		44
3.3 Further pulley systems		49
3.4 Symmetry		57
4 Friction		60
4.1 Coefficients of friction		60
4.2 Further applications		64
4.3 Why does the wheel work?		68
5 Differential equations: linearity and SHM		71
5.1 Guessing solutions		71
5.2 How many solutions?		74
5.3 Linearity		77
5.4 The SHM equation		81

vi	<i>Contents</i>	
6	Springs and oscillations	85
6.1	Force in a spring	85
6.2	A basic example	88
6.3	Further spring problems	94
	Part two: Models with Difference Equations	103
7	Difference equations	105
7.1	Introductory example	105
7.2	Difference equations — basic ideas	109
7.3	Constant solutions and fixed points	114
7.4	Iteration and cobweb diagrams	118
8	Linear difference equations in finance and economics	126
8.1	Linearity	127
8.2	Interest and loan repayment	133
8.3	The cobweb model of supply and demand	138
8.4	National income: ‘acceleration models’	142
9	Non-linear difference equations and population growth	146
9.1	Linear models for population growth	146
9.2	Restricted growth — non-linear models	152
9.3	A computer experiment	157
9.4	A coupled model of a measles epidemic	164
9.5	Linearizing non-linear equations	170
10	Models for population genetics	177
10.1	Some background genetics	177
10.2	Random mating with equal survival	185
10.3	Lethal recessives, selection and mutation	193
	Part three: Models with Differential Equations	201
11	Continuous growth and decay models	203
11.1	First-order differential equations	203
11.2	Exponential growth	212
11.3	Restricted growth	218
11.4	Exponential decay	227
12	Modelling heat flow	232
12.1	Newton’s model of heating and cooling	232
12.2	More physics in the model	237
12.3	Conduction and insulation	241
12.4	Insulating a pipe	249
13	Compartment models of mixing	257
13.1	A mixing problem	257
13.2	Modelling pollution in a lake	265
13.3	Modelling heat loss from a hot water tank	270

	<i>Contents</i>	vii
	Part four: Further Mechanics	275
14	Motion in a fluid medium	277
14.1	Some basic fluid mechanics	277
14.2	Archimedes' Principle	282
14.3	Falling sphere with Stokes' resistance	286
14.4	Falling sphere with velocity-squared drag	290
15	Damped and forced oscillations	295
15.1	Constant-coefficient differential equations	295
15.2	Damped oscillations	302
15.3	Forced harmonic motion	311
16	Motion in a plane	318
16.1	Kinematics in a plane	318
16.2	Motion down an inclined plane	326
16.3	Projectiles	331
17	Motion on a circle	336
17.1	Kinematics on a circle	336
17.2	Uniform circular motion	343
17.3	The pendulum and linearization	348
	Part five: Coupled Models	353
18	Models with linear interactions	355
18.1	Two-compartment mixing	355
18.2	Solving constant-coefficient equations	360
18.3	A model for detecting diabetes	366
18.4	Nutrient exchange in the placenta	373
19	Non-linear coupled models	379
19.1	Predator–prey interactions	379
19.2	Phase-plane analysis	384
19.3	Models of combat	389
19.4	Epidemics	394
	<i>References</i>	399
	<i>Index</i>	403