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978-0-521-44474-3 - Generalized Topological Degree and Semilinear Equations
Wolodymyr V. Petryshyn
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This book describes the construction of the generalized topological degree for densely defined and not necessarily continuous A-proper operators, and presents important applications. A-proper mappings arise naturally in the solution to an equation in infinite dimensional space via the finite dimensional approximation. The theory subsumes classical theory involving compact vector fields as well as the more recent theories of condensing vector fields and strongly monotone and strongly accretive maps.

The book begins with an outline of Brouwer degree theory and a description of some basic constructive results. Using these tools, the author defines the generalized topological degree for densely defined A-proper mappings, gives applications to the solvability of an important class of semilinear abstract and differential equations, and discusses global bifurcation results. These abstract results are then applied to boundary value problems of ODEs and PDEs with general nonlinearities, problems that are intractable under any other existing theory.

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To my wife, Arcadia

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Preface

In this monograph we develop the generalized degree theory for densely defined A-proper mappings, and then use it to study the solvability (sometimes constructive) and the structure of the solution set of the important class of semilinear abstract and differential equations

$$Lx - N(x) = y, \quad x \in D(L), \ y \in Y, \tag{0.1}$$

where X and Y are Banach spaces, $L: D(L) \subseteq X \to Y$ is in general an unbounded Fredholm map of index $i(L) \geq 0$ with nullspace $N(L) \neq \{0\}$, and N is a nonlinearity such that $L - N$ is A-proper. As is well known, this is a very general class of semilinear abstract, ordinary and partial differential equations which – unlike classical theory – does not require the partial inverse of L to be compact, or N to be compact or even condensing. Thus, in addition to classical problems, by the A-proper mapping theory one can solve certain differential boundary value problems for ODEs and PDEs which cannot be solved by any other existing abstract theory.

The first part of Chapter 1 contains an outline of the Brouwer degree theory, since it is needed in Chapter 2 for the definition of generalized topological degree for densely defined A-proper mappings, proofs of its basic properties, and some existence theorems. The second part introduces the notion of the Leray–Schauder degree and some needed properties; it also contains the basic constructive results for equations involving bounded linear A-proper maps.

Chapter 2 presents the theory of the generalized degree for densely defined A-proper maps, as well as some of its application to the solvability of (0.1). Also presented are global bifurcation results for the equation

$$L(\lambda)(x) - N(\lambda, x) = 0, \quad (\lambda, x) \in \mathbb{R} \times X, \tag{0.2}$$

involving A-proper maps $L(\lambda) \in L(X, Y)$ and $L(\lambda) - N(\lambda, \cdot)$.

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In Chapter 3, the abstract results from Chapter 2 are applied to the solvability (sometimes constructive) of periodic ODEs of order $n \geq 2$. In particular, the generalized Lienard equation, equations describing the dynamics of wires, and equations of order $n > 2$ from elasticity and mechanics are studied.

Chapter 4 contains further existence theorems for equation (0.1), under various conditions on N, that are useful in the study of ODEs and PDEs. Some related topics are also discussed, including study of the structure of the solution set of (0.1).

In Chapter 5 the theory of A-proper maps is applied to obtain existence and structure results for semilinear elliptic PDEs of order 2 and $2m$, equations in which neither the partial inverse of L nor the nonlinearity N is compact or condensing and where N depends on the highest-order derivatives. Some of our results are new, and some properly extend important results of other authors. Each chapter is finished with appropriate notes and problems.

The appendix contains, for the convenience of the reader, some known results used in this monograph.

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My thanks also go to my wife, Arcadia, and to those of my friends who have continuously encouraged me to write this monograph.

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W. V. Petryshyn

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