Vibrations and waves in physics

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Contents

1

Free vibrations

Most physical systems possess certain properties which enable them, under suitable conditions, to vibrate; we shall examine a few examples in chapter 2. But in order to discover the essential features of vibrational behaviour, we first consider a 'model system': an imaginary prototype system which possesses those properties which are necessary for vibrational behaviour, and no others. This is a well-tried procedure in physics. The basic idea is that, after examining in detail the behaviour of the model, we shall be able to recognize, in real, complicated systems, features which can lead to vibrational behaviour.

The model system in this case is a mass m attached to one end of a light helical spring whose other end is fixed (fig. 1.1). In equilibrium we suppose that adjacent coils of the spring are not in contact, so that there is scope for it to be compressed as well as stretched. We choose to ignore all forces not due to the elasticity of the spring: gravity, friction and viscosity are all 'switched off'.

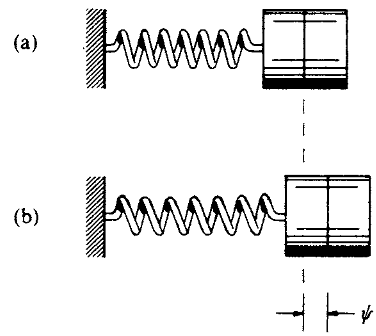

Fig. 1.1 (a) The prototype vibrator in equilibrium. (b) The mass is instantaneously displaced a distance ψ to the right of its equilibrium position.

If the system has been disturbed at some earlier time, it may not be in equilibrium. The mass may be at some position a distance ψ to the right of its equilibrium position, as in fig. 1.1(b). In that case the spring will exert a force towards the left. If the displacement is to the left, the force will act to the right. In either case the magnitude of the force will increase as the size of the displacement increases: the mass always experiences a *return force* which tends to change the displacement ψ towards the value zero.

It is easy to see that the free motion of this system takes the form of a vibration. The mass is given an acceleration

$$\ddot{\psi} \equiv \frac{d^2\psi}{dt^2}$$

which is determined by Newton's second law

$$m\ddot{\psi} = F_s \tag{1.1}$$

where F_s is the spring force. It will thus have acquired a certain velocity, and a corresponding momentum, by the time it reaches its equilibrium position, and so it will overshoot. Now the mass is acted on by a return force in the opposite direction; it is decelerated, brought to rest, and accelerated back to its equilibrium position where it overshoots again. The direction of the displacement continually alternates.

It is clear that both the elasticity or 'stiffness' of the spring and the inertial property of the mass are necessary for vibrational motion: the stiffness ensures that the mass tries to return to its equilibrium position, whereas the inertia makes it overshoot. We shall find that all vibrational phenomena depend on the existence of a pair of quantities analogous to stiffness and inertia.

1.1. Harmonic motion

The equation of motion (1.1) is a second-order differential equation from which we wish to find an expression giving ψ as a function of the time t. The equation is too vague as it stands, however: we can solve it only if we know exactly how F_s varies with ψ.

In order to get quantitative results, we shall make the simplest possible assumption, that F_s is *proportional to* ψ for the particular spring we are dealing with. We write

$$F_s = -s\psi \tag{1.2}$$

where s is a positive constant known as the spring constant, or the *stiffness*. With this assumption, our system now possesses all the properties of the imaginary object known to physics as the Harmonic Oscillator.

The equation of motion (1.1) now becomes

$$m\ddot{\psi} = -s\psi \tag{1.3}$$

In order to make the forthcoming results easier to carry over to other vibrating systems, we shall write this equation in a standard form

$$\ddot{\psi} + \omega_0^2\psi = 0 \tag{1.4}$$

which contains a new, positive quantity

$$\omega_0 \equiv |(s/m)^{1/2}| \tag{1.5}$$

It is easy to verify, by differentiation and substitution, that (1.4) is satisfied by an expression of the form

$$\psi(t) = A\cos(\omega_0 t + \phi) \tag{1.6}$$

where A is any constant length and ϕ is any constant angle. When a quantity depends on time in this way it is said to vary harmonically. A vibration in which ψ varies harmonically is known as *harmonic motion*; it occurs for a mass on a spring only when the spring obeys (1.2).

The controlling quantity in (1.6) is the *phase angle* $\omega_0 t + \phi$, sometimes called simply the phase. The phase angle increases uniformly with time; values of any angle differing by an integral multiple of 2π are physically indistinguishable, however. Thus harmonic motion is *periodic*, repeating itself endlessly in a sequence of identical *cycles*. All measurable quantities, such as the displacement, speed, direction of travel and acceleration of the mass, recur whenever the phase angle increases by 2π. This will happen at times separated by the interval τ given by $\omega_0\tau = 2\pi$ (fig. 1.2). This characteristic time interval is called the *period* of the vibration.

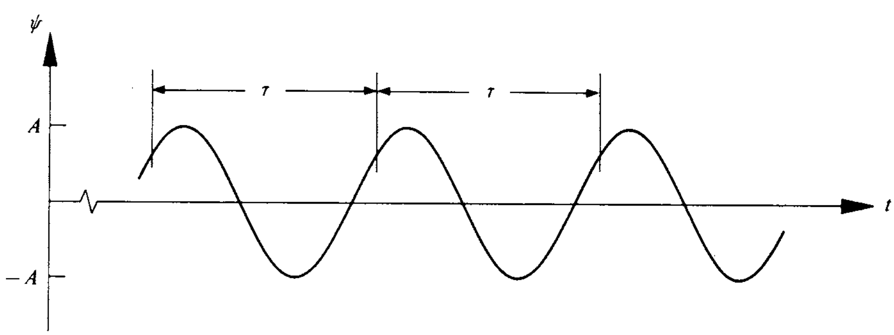

Fig. 1.2 How ψ varies with time during harmonic vibration. Identical events are separated by time intervals τ. The origin $t = 0$ can be placed at any convenient stage in the cycle of events.

Of the two constants in the phase angle, ω_0 is fixed by (1.5) but any value of ϕ, the *phase constant*, will give an acceptable solution of (1.4). Changing the value of ϕ merely makes all events in the cycle happen earlier or later by the same amount, without affecting the sequence of events within any cycle. Again, retarding or advancing the action by any whole number of periods will produce no observable difference. Thus we can increase or decrease ϕ by any whole multiple of 2π without changing anything physically.

During a cycle of vibration, ψ takes on all values between the limits $\pm A$; we call A the *amplitude*. The number of cycles per unit time,

$$\nu_0 = \omega_0/2\pi$$

is the *frequency*. If the time is measured in seconds, ν_0 is quoted in hertz (Hz); a vibration at 5000 cycles per second, for example, has a frequency of 5 kHz.

The quantity ω_0 clearly has the same dimensions as ν_0. Since $\omega_0 t$ appears in (1.6) as an angle, we shall usually call ω_0 the *angular frequency* to distinguish it from ν_0. Although ν_0 is easier to measure, ω_0 makes for tidier formulas containing fewer factors of 2π. To distinguish the two quantities further, we shall measure ω_0 in inverse seconds (s^{-1}) rather than hertz.†

Boundary conditions. Equation (1.6), being the solution of a second-order differential equation (1.4), correctly contains two arbitrary constants. Any pair of values of A and ϕ will describe a vibration which *can* be executed by the mass and spring provided. In practice, however, we shall be dealing with a particular vibration whose details have been determined by some other physical conditions: usually the method used to set the vibration going in the first place. These *boundary conditions* will fix the values of A and ϕ that must be used for that particular vibration.

If, for example, the mass was originally held steady at some distance A_1 to the right of its equilibrium position, and then released at time $t = 0$, we could say that

$$\begin{aligned} \psi(0) &= A \cos\phi = A_1 \\ \dot{\psi}(0) &= -\omega_0 A \sin\phi = 0 \end{aligned} \qquad (1.7)$$

† One might use rad s^{-1} for angular frequencies. The radian, however, being dimensionless, is not so much a unit as a signal saying 'angle'. It is more convenient to omit the radian when discussing vibrations, as we shall be cancelling angular frequencies with other quantities measured in s^{-1} and having nothing to do with angles.

where $\dot{\psi}$ means $d\psi/dt$. These two boundary conditions (which are actually *initial conditions* in this case) are sufficient to fix A and ϕ. The second condition tells us that ϕ is either 0 or π; we reject the latter alternative because the first condition makes $\cos\phi$ positive. Thus $A = A_1$ and (1.6) becomes

$$\psi(t) = A_1 \cos \omega_0 t \tag{1.8}$$

Different starting arrangements would lead to different answers for A and ϕ.

Phase differences. In the example above we chose to label the moment at which the vibration was started as $t = 0$. This choice assumes that we are measuring time with a stopwatch which we start at the instant when the mass is released. There is no reason why we should not use an ordinary clock and start the vibration at some different time t_0; but by choosing the starting time to be zero we have been able to arrange that ϕ is zero also. As we saw above, having a different value for ϕ would merely advance or retard all the action by the same amount in time.

The real significance of the phase constant becomes apparent when we are dealing with two or more vibrations of the same frequency. Some examples are shown in fig. 1.3. In each case one of the vibrations is of the kind described by (1.8); that is, we have started it in the way described and have chosen $t = 0$ as the starting time.

The second vibration in fig. 1.3(a) has a different amplitude A_2, but it again has zero phase constant. The two displacements vary exactly in step with each other, and are always in the ratio A_2/A_1. We say that these two vibrations are *in phase*.

Under all other circumstances we say that the vibrations are *out of phase*. In fig. 1.3(b) the second vibration has $\phi > 0$, and all events such as passing the equilibrium point from left to right, or reaching the point of maximum positive ψ, happen earlier for this vibration than for the other. In this case we say that the second vibration leads in phase by ϕ, or has a *phase advance* of ϕ relative to the first. When the second vibration has $\phi < 0$ as in fig. 1.3(c), it is said to lag in phase by $|\phi|$, or to have a *phase lag* of $|\phi|$.

Clearly a phase lag greater than 180° is equivalent to a phase lead of less than 180°, and it is usually more convenient to quote the smaller value. The two special cases $\phi = \pm\pi$ are of course equivalent; vibrations such as those in fig. 1.3(d) with a 180° phase difference are said to be *in antiphase*. If $|\phi| = \frac{1}{2}\pi$ they are *in quadrature*.

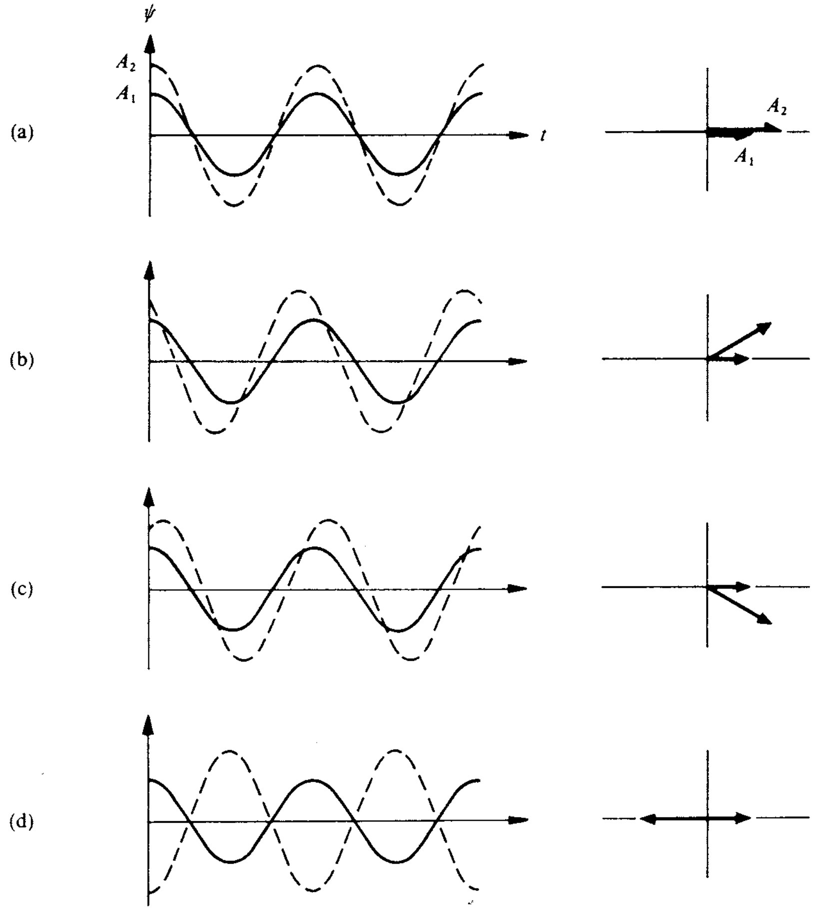

Fig. 1.3 Phase differences. In each diagram the vibration with amplitude A_1 has a phase constant of zero. The vibration with amplitude A_2 has (a) $\phi = 0$; (b) $\phi > 0$; (c) $\phi < 0$; (d) $\phi = \pm\pi$. Vector diagrams representing these vibrations are shown on the right.

Choosing our zero of time in a different way would have led to a non-zero phase constant for the first vibration in these examples. The phase constant of the other vibration would have been increased or decreased by the same amount, however, and the *phase difference* would have been unaffected.

Vector diagrams. For handling two or more harmonic vibrations of the same frequency, a geometrical method is helpful. We have already noted that the phase angle $\omega_0 t + \phi$ increases uniformly with time as the vibration takes place. The displacement at any moment t is proportional to the cosine of this angle. We can therefore generate $\psi(t)$ by letting a radius vector of length A rotate anticlockwise, as in fig. 1.4, and projecting it on to some fixed axis. An axis through the origin is the most convenient. The radius vector should make an angle ϕ with the axis at time $t=0$, and should rotate with angular speed ω_0. The rotating vector is sometimes called a *phasor.*

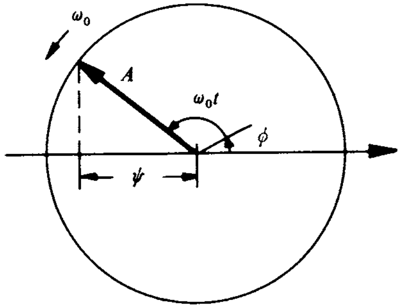

Fig. 1.4 The rotating vector which generates $\psi(t)$. At time $t=0$ the vector makes an angle ϕ (anticlockwise) with the reference axis. At other times it makes an angle $\omega_0 t+\phi$. Its projection on the reference axis is $\psi(t)$.

A second vibration, with a different amplitude and phase constant, is represented by a vector of a different length, rotating at a fixed angle (the phase difference) to the first. This angle is measured in an anticlockwise sense from the first vector to the second if the second vibration leads the first; conversely, a phase lag is measured clockwise.

Since the vibrations have a common frequency, we shall usually be more interested in amplitudes and phase constants. Their values can be specified by means of a static diagram showing the rotating vectors in their $t=0$ positions. Vector diagrams for the four examples in fig. 1.3 are shown beside the $\psi-t$ plots.

Velocity and acceleration. As an example of the use of vector diagrams, we illustrate the phase relationships between the displacement, the velocity and the acceleration of the mass during harmonic vibration. The velocity can be obtained as a function of time by differentiating (1.6) with respect to t; we find

$$\begin{aligned}\dot{\psi}(t) &= -\omega_0 A \sin(\omega_0 t+\phi)\\ &= \omega_0 A \cos(\omega_0 t+\phi+\tfrac{1}{2}\pi)\end{aligned} \tag{1.9}$$

A second differentiation gives the acceleration

$$\begin{aligned}\ddot{\psi}(t) &= -\omega_0^2 A \cos(\omega_0 t+\phi)\\ &= \omega_0^2 A \cos(\omega_0 t+\phi+\pi)\end{aligned}$$

These expressions show that $\dot{\psi}$ and $\ddot{\psi}$, like ψ, vary harmonically, and that the frequency is the same in all three cases. The amplitude of $\dot{\psi}$ (the maximum speed reached by the mass) is $\omega_0 A$, and that of $\ddot{\psi}$ is $\omega_0^2 A$. The velocity, however, leads the displacement in phase by 90°; the acceleration leads the velocity by a further 90°, bringing it into antiphase with the displacement, as is necessary if (1.3) is to be obeyed.

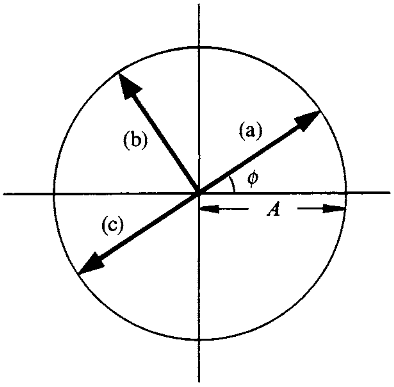

Fig. 1.5 Vectors showing the relative phases of (a) the displacement, (b) the velocity, and (c) the acceleration, for a vibration with amplitude A and phase constant ϕ. The velocity amplitude is $\omega_0 A$ and the acceleration amplitude is $\omega_0^2 A$.

In vector diagram terms (fig. 1.5) each differentiation with respect to t can be seen as a multiplication of the length of the rotating vector by ω_0, together with an anticlockwise rotation through 90°. The vectors drawn in fig. 1.5 are all lengths. On other occasions it may be more convenient to choose vectors which represent velocities, accelerations or forces; these quantities should not be mixed in the same diagram, however.

Energy. As usual in mechanical systems, two kinds of energy are present. When the mass is moving with speed $|\dot{\psi}|$ in either direction, its kinetic energy is

$$T = \tfrac{1}{2} m \dot{\psi}^2 \tag{1.10}$$

When the spring is stretched or compressed by an amount $|\psi|$, it stores potential energy

$$V = \tfrac{1}{2} s \psi^2 \tag{1.11}$$

The total energy

$$W = T + V = \tfrac{1}{2} m \dot{\psi}^2 + \tfrac{1}{2} s \psi^2 \tag{1.12}$$

remains constant during the vibration, since all dissipative forces like friction and viscosity are assumed to be absent. Thus

$$\frac{dW}{dt} = 0 \tag{1.13}$$

Using (1.12) we obtain

$$m\dot{\psi}\ddot{\psi} + s\psi\dot{\psi} = 0$$

$$m\ddot{\psi} + s\psi = 0$$

The last equation is just (1.3), reached by a new route.

To discover how T and V vary individually with time, we substitute (1.9) into (1.10), and (1.6) into (1.11), to find

$$T = \tfrac{1}{2}m\omega_0^2 A^2 \sin^2(\omega_0 t + \phi)$$

$$V = \tfrac{1}{2}sA^2 \cos^2(\omega_0 t + \phi)$$

These are plotted in fig. 1.6. The value of their constant sum is

$$W = \tfrac{1}{2}m(\omega_0 A)^2 = \tfrac{1}{2}sA^2$$

since $s = m\omega_0^2$ by (1.5). For a given mass and spring, the total energy is proportional to the square of the amplitude, but does not depend on the phase constant.

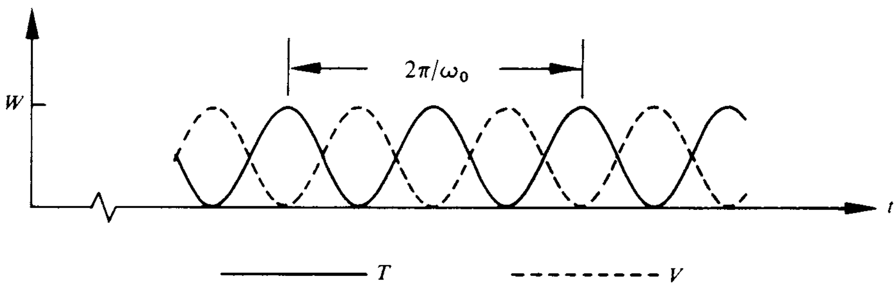

Fig. 1.6 Variation of kinetic energy T and potential energy V during harmonic vibration. The total energy W is constant.

We can think of the vibration as the repeated transfer of a fixed amount of energy from the mass to the spring and back again, twice in each cycle. When the spring is stretched or compressed its maximum distance ($\psi = \pm A$) the mass comes momentarily to rest and the kinetic energy vanishes. At that moment all the energy of the system is stored in the spring as potential energy. When the mass is passing through $\psi = 0$ (in either direction) it has its maximum speed $\omega_0 A$ and contains the entire energy of the system, since the spring is neither stretched nor compressed. At other points in the cycle there is a variable mixture of kinetic and potential energy, but their sum never changes.

Summary. The free motion of the model system is a vibration because the system possesses the two essential properties of stiffness and inertia. In the special case of a spring which exerts a return force proportional to the displacement of the mass, the vibration is harmonic. For a given mass and a given spring the frequency is fixed, but the amplitude and the phase constant depend on boundary conditions.

The velocity and the acceleration of the mass also vary harmonically, with the same frequency as the displacement. Because displacement and velocity are in quadrature, the energy of the system flows back and forth between the mass and the spring twice in each cycle.

1.2. Alternative mathematics for harmonic motion

There are several alternative ways of writing the solution (1.6). Each has its own special advantages, and in the rest of the book we shall adopt whichever one is most suitable for the particular purpose in hand. From now on we shall refer to (1.6) as form A; here we derive, with little further comment, three other versions which we identify for convenience as forms B, C and D.

In discussing the various vibrational systems introduced in the next chapter it is not necessary to write down the solution at all. You may therefore prefer to leave this section until later.

Form B. Expanding the cosine in (1.6) gives immediately

$$\begin{aligned}\psi(t) &= A\cos\phi\cos\omega_0 t - A\sin\phi\sin\omega_0 t \\ &= B_p\cos\omega_0 t + B_q\sin\omega_0 t\end{aligned} \tag{1.14}$$

We call this form B. Information about the amplitude and the phase constant of the vibration is conveyed by means of two new constants

$$B_p \equiv A\cos\phi$$

$$B_q \equiv -A\sin\phi$$

each of which can be either positive or negative. These constants will be fixed by the boundary conditions, as were A and ϕ. Thus, in the example considered in the previous section we would find $B_p = A_t$ and $B_q = 0$: comparison of (1.8) with (1.14) gives the same result directly.

It is worth noting here that a second vibration of amplitude A_2, in quadrature with the previous one, has $B_p = 0$ and $B_q = \pm A_2$, and so takes the simple form

$$\psi(t) = \pm A_2 \sin \omega_0 t$$

Here the plus sign means that the second vibration lags the first, and *vice versa*. By adopting a vibration with $\phi = 0$ as a standard of phase (as in fig. 1.3) we can use the suffixes p ('phase') and q ('quadrature') as memory aids.

Form C. We can always try to solve a differential equation like (1.4) by substituting a trial expression of the form

$$\psi = C\,\mathrm{e}^{pt}$$

In the present case we find that such a solution is acceptable if the relation

$$p^2 = -\omega_0^2$$

is satisfied. This will be so for two values of p, given by $\pm\mathrm{i}\omega_0$. The general solution may therefore be written as a linear combination

$$\psi(t) = C \exp(\mathrm{i}\omega_0 t) + C' \exp(-\mathrm{i}\omega_0 t) \tag{1.15}$$

in which the constants C and C' are both complex.

This solution is actually too general for our purpose. It has four arbitrary constants (Re C, Im C, Re C' and Im C') whereas the boundary conditions can only cope with two. It is easy to see what is wrong with (1.15) as it stands. In order to represent a single physical displacement, ψ may be either real or imaginary, but must not be complex. The expression on the right of (1.15) can be made real by insisting that the two terms are complex conjugates of each other, that is

$$[C' \exp(-\mathrm{i}\omega_0 t)]^* = C \exp(\mathrm{i}\omega_0 t)$$

The complex conjugate of a product of two quantities is the product of their complex conjugates, and so

$$C'^* \exp(\mathrm{i}\omega_0 t) = C \exp(\mathrm{i}\omega_0 t)$$

$$C' = C^*$$

Form C now appears as

$$\psi(t) = C \exp(\mathrm{i}\omega_0 t) + C^* \exp(-\mathrm{i}\omega_0 t) \tag{1.16}$$

Now there are only two arbitrary constants (Re C and Im C). By expanding the exponentials in (1.16) in terms of sines and cosines with the aid of de Moivre's theorem

$$e^{ix} = \cos x + i \sin x$$

and comparing the result with (1.14), we obtain the relation between these new constants and the previous ones,

$$\begin{aligned} \text{Re}\, C &= \tfrac{1}{2}B_p = \tfrac{1}{2}A \cos \phi \\ \text{Im}\, C &= -\tfrac{1}{2}B_q = \tfrac{1}{2}A \sin \phi \end{aligned} \tag{1.17}$$

Form D. Since we have arranged that the expression on the right of (1.16) is real, it must be possible to write

$$\begin{aligned} \psi(t) &= \text{Re}\,[C \exp(i\omega_0 t)] + \text{Re}\,[C^* \exp(-i\omega_0 t)] \\ &= 2\,\text{Re}\,[C \exp(i\omega_0 t)] \end{aligned} \tag{1.18}$$

By defining a new complex constant $D \equiv 2C$, we reach form D,

$$\psi(t) = \text{Re}\,[D \exp(i\omega_0 t)] \tag{1.19}$$

We shall call D the *complex amplitude* of the vibration. Like C, however, it contains information about the phase constant.

The arbitrary constants of forms A and D are linked through the relations

$$\begin{aligned} \text{Re}\, D &= A \cos \phi \\ \text{Im}\, D &= A \sin \phi \end{aligned} \tag{1.20}$$

which follow immediately from (1.17). For a vibration started as before with $\phi = 0$, D is purely real; for a vibration in quadrature with that one ($\phi = \pm\tfrac{1}{2}\pi$) D is imaginary.

Form D provides an extremely powerful way of handling harmonic motion. Its chief merit is the ease with which it can be differentiated and integrated. To find $\dot{\psi}$, for example, we could differentiate (1.16) with respect to t and proceed as we did in (1.18) and (1.19). The result

$$\dot{\psi}(t) = \text{Re}\,[i\omega_0 D \exp(i\omega_0 t)]$$

is, however, just what we would get if we first differentiated the complex function $D \exp(i\omega_0 t)$ and *then* took the real part. The same is true of the acceleration

$$\ddot{\psi}(t) = \text{Re}\,[-\omega_0^2 D \exp(i\omega_0 t)] \tag{1.21}$$

At each differentiation we merely have to multiply the complex function by $i\omega_0$.

It is instructive to examine the behaviour of the controlling function $D \exp(i\omega_0 t)$ on an Argand diagram. First we write the complex amplitude in conventional modulus-and-argument form,

$$D = A\cos\phi + i(A\sin\phi) = A\exp(i\phi)$$

The whole function then becomes

$$D\exp(i\omega_0 t) = A\exp[i(\omega_0 t + \phi)]$$

On the Argand diagram (fig. 1.7) this is represented by a radius vector of length A rotating anticlockwise with angular speed ω_0. Its initial ($t = 0$) position makes an angle ϕ measured anticlockwise from the real axis.

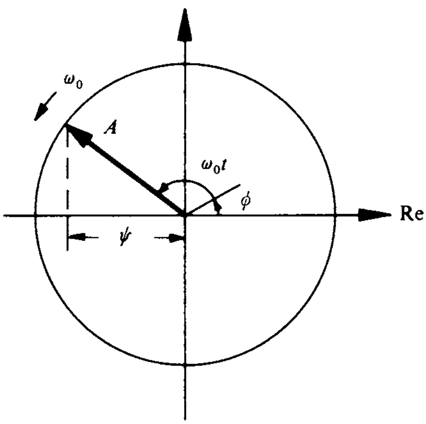

Fig. 1.7 Argand diagram representing $\psi(t)$. The real axis is equivalent to the reference axis of fig. 1.4.

This is, of course, the same rotating vector as the one in fig. 1.4. In complex number terminology we describe ψ as the projection on the real axis.

In this book form D will always be written as in (1.19); but it is common practice to write ψ *equal to* a complex function such as $D\exp(i\omega_0 t)$, leaving implicit the instruction to take the real part. It will be obvious that this is being done whenever a simple physical quantity such as a length appears to be complex.

Summary. We collect together for reference purposes the four forms of the solution,

$$\begin{aligned} \psi(t) &= A\cos(\omega_0 t + \phi) \\ \psi(t) &= B_p\cos\omega_0 t + B_q\sin\omega_0 t \\ \psi(t) &= C\exp(i\omega_0 t) + C^*\exp(-i\omega_0 t) \\ \psi(t) &= \mathrm{Re}[D\exp(i\omega_0 t)] \end{aligned} \tag{1.22}$$

Each version contains two real constants whose values can be adjusted to fit boundary conditions.

It will sometimes be convenient to change from one form to another. The simplest relations connecting the four pairs of constants are

$$\begin{aligned} A \cos\phi &= B_p = 2 \,\mathrm{Re}\, C = \mathrm{Re}\, D \\ A \sin\phi &= -B_q = 2 \,\mathrm{Im}\, C = \mathrm{Im}\, D \end{aligned} \tag{1.23}$$

Other relationships, such as the expressions giving A and ϕ individually in terms of the other constants, can readily be obtained from (1.23).

Problems

1.1 If the system shown in fig. 1.1 has $m = 0.010$ kg and $s = 36$ N m^{-1}, calculate (a) the angular frequency, (b) the frequency, and (c) the period.

1.2 For the same vibrator as in problem 1.1, at time $t = 0$, the mass is observed to be displaced 50 mm to the right of its equilibrium position and to be moving to the right at a speed 1.7 m s^{-1}. Calculate (a) the amplitude, (b) the phase constant, and (c) the energy.

1.3 An identical system is set into vibration with the same amplitude as the vibrator in problem 1.2, but with a phase advance of 90°. Calculate (a) the displacement, and (b) the velocity of this second vibrator at time $t = 0$. (c) At what time will it next come to rest?

1.4 The system shown at rest in fig. 1.1(a) could be set into vibration by giving the mass a sudden momentum impulse to the left: by tapping it with a hammer, for example. If the magnitude of the impulse is p_1 and it is given at time $t = 0$, find (a) the amplitude and (b) the phase constant of the ensuing motion.

1.5 The system shown at rest in fig. 1.1 (a) could be set into motion by giving it an initial displacement A_1 *and* an initial velocity v_1 (both to the right, say). Assuming that the motion is started in this way at time $t = 0$, show that the amplitude A and the phase constant ϕ are given by

$$A = [A_1^2 + (v_1/\omega_0)^2]^{1/2}$$
$$\tan\phi = -v_1/A_1\omega_0$$

1.6 Calculate (a) the amplitude, (b) the phase constant, and (c) the complex amplitude, for the vibration given by

$$\psi = (10 \text{ mm}) \cos \omega_0 t + (17 \text{ mm}) \sin \omega_0 t$$

1.7 During a vibration with a frequency of 50 Hz, the displacement is observed to be 30 mm at time $t = 0$, and -14 mm at $t = 12$ ms. Find the complex amplitude.

1.8 Calculate the maximum acceleration (in units of g) of a pickup stylus reproducing a frequency of 16 kHz, with an amplitude of 0.01 mm.