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0521441773 - Affine Differential Geometry: Geometry of Affine Immersions - Katsumi Nomizu and Takeshi Sasaki

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