

> This is a revised edition of a classic and highly regarded book, first published in 1981, giving a comprehensive survey of the intensive research and testing of general relativity that has been conducted over the last three decades. As a foundation for this survey, the book first introduces the important principles of gravitation theory, developing the mathematical formalism that is necessary to carry out specific computations so that theoretical predictions can be compared with experimental findings. A completely up-to-date survey of experimental results is included, not only discussing Einstein's "classical" tests, such as the deflection of light and the perihelion shift of Mercury, but also new solar system tests, never envisioned by Einstein, that make use of the high precision space and laboratory technologies of today. The book goes on to explore new arenas for testing gravitation theory in black holes, neutron stars, gravitational waves and cosmology. Included is a systematic account of the remarkable "binary pulsar" PSR 1913+16, which has yielded precise confirmation of the existence of gravitational waves.

> The volume is designed to be both a working tool for the researcher in gravitation theory and experiment, as well as an introduction to the subject for the scientist interested in the empirical underpinnings of one of the greatest theories of the twentieth century.

### Comments on the previous edition:

"consolidates much of the literature on experimental gravity and should be invaluable to researchers in gravitation" Science

"a concise and meaty book . . . and a most useful reference work . . . researchers and serious students of gravitation should be pleased with it" Nature



Theory and Experiment in Gravitational Physics

Revised Edition



# THEORY AND EXPERIMENT IN GRAVITATIONAL PHYSICS

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Revised Edition





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To Leslie



### **Contents**

	Preface to Revised Edition	<i>page</i> xiii
	Preface to First Edition	xv
1	Introduction	1
2	The Einstein Equivalence Principle and the	
	Foundations of Gravitation Theory	13
2.1	The Dicke Framework	16
2.2	Basic Criteria for the Viability of a Gravitation Theory	18
2,3	The Einstein Equivalence Principle	22
2.4	Experimental Tests of the Einstein Equivalence Principle	24
2.5	Schiff's Conjecture	38
2.6	The THεμ Formalism	45
3	Gravitation as a Geometric Phenomenon	67
3.1	Universal Coupling	67
3.2	Nongravitational Physics in Curved Spacetime	68
3.3	Long-Range Gravitational Fields and the Strong	
	Equivalence Principle	79
4	The Parametrized Post-Newtonian Formalism	86
4.1	The Post-Newtonian Limit	87
4.2	The Standard Post-Newtonian Gauge	96
4.3	Lorentz Transformations and the PPN Metric	99
4.4	Conservation Laws in the PPN Formalism	105
5	Post-Newtonian Limits of Alternative Metric	
	Theories of Gravity	116
5.1	Method of Calculation	116
5.2	General Relativity	121
5.3	Scalar-Tensor Theories	123
5.4	Vector-Tensor Theories	126
5.5	Bimetric Theories with Prior Geometry	130
5.6	Stratified Theories	135
5.7	Nonviable Theories	138

ix



	Contents	Х
6	Equations of Motion in the PPN Formalism	142
6.1	Equations of Motion for Photons	143
6.2	Equations of Motion for Massive Bodies	144
6.3	The Locally Measured Gravitational Constant	153
6.4	N-Body Lagrangians, Energy Conservation, and the Strong	
	Equivalence Principle	158
6.5	Equations of Motion for Spinning Bodies	163
7	The Classical Tests	166
7.1	The Deflection of Light	167
7.2	The Time-Delay of Light	173
7.3	The Perihelion Shift of Mercury	176
8	Tests of the Strong Equivalence Principle	184
8.1	The Nordtvedt Effect and the Lunar Eötvös Experiment	185
8.2	Preferred-Frame and Preferred-Location Effects:	
	Geophysical Tests	190
8.3	Preferred-Frame and Preferred-Location Effects: Orbital Tests	200
8.4	Constancy of the Newtonian Gravitational Constant	202
8.5	Experimental Limits on the PPN Parameters	204
9	Other Tests of Post-Newtonian Gravity	207
9.1	The Gyroscope Experiment	208
9.2	Laboratory Tests of Post-Newtonian Gravity	213
9.3	Tests of Post-Newtonian Conservation Laws	215
10	Gravitational Radiation as a Tool for Testing Relativistic Gravity	221
10.1	Speed of Gravitational Waves	223
10.2	Polarization of Gravitational Waves	227
10.3	Multipole Generation of Gravitational Waves and Gravitational	
	Radiation Damping	238
11	Structure and Motion of Compact Objects in	
	Alternative Theories of Gravity	255
11.1	Structure of Neutron Stars	257
11.2	Structure and Existence of Black Holes	264
11.3	The Motion of Compact Objects: A Modified EIH Formalism	266
12	The Binary Pulsar	283
12.1	Arrival-Time Analysis for the Binary Pulsar	287
12.2	The Binary Pulsar According to General Relativity	303
12.3	The Binary Pulsar in Other Theories of Gravity	306
13	Cosmological Tests	310
13.1	Cosmological Models in Alternative Theories of Gravity	312
13.2	Cosmological Tests of Alternative Theories	316



	Contents	xi
14	An Update	320
14.1	The Einstein Equivalence Principle	320
14.2	The PPN Framework and Alternative Metric Theories of	
	Gravity	331
14.3	Tests of Post-Newtonian Gravity	332
14.4	Experimental Gravitation: Is there a Future?	338
14.5	The Rise and Fall of the Fifth Force	341
14.6	Stellar-System Tests of Gravitational Theory	343
14.7	Conclusions	352
	References	353
	References to Chapter 14	371
	Index	375



# Preface to the Revised Edition

Since the publication of the first edition of this book in 1981, experimental gravitation has continued to be an active and challenging field. However, in some sense, the field has entered what might be termed an Era of Opportunism. Many of the remaining interesting predictions of general relativity are extremely small effects and difficult to check, in some cases requiring further technological development to bring them into detectable range. The sense of a systematic assault on the predictions of general relativity that characterized the "decades for testing relativity" has been supplanted to some extent by an opportunistic approach in which novel and unexpected (and sometimes inexpensive) tests of gravity have arisen from new theoretical ideas or experimental techniques, often from unlikely sources. Examples include the use of laser-cooled atom and ion traps to perform ultra-precise tests of special relativity, and the startling proposal of a "fifth" force, which led to a host of new tests of gravity at short ranges. Several major ongoing efforts continued nonetheless, including the Stanford Gyroscope experiment, analysis of data from the Binary Pulsar, and the program to develop sensitive detectors for gravitational radiation observatories.

For this edition I have added chapter 14, which presents a brief update of the past decade of testing relativity. This work was supported in part by the National Science Foundation (PHY 89-22140).

Clifford M. Will 1992



# Preface to First Edition

For over half a century, the general theory of relativity has stood as a monument to the genius of Albert Einstein. It has altered forever our view of the nature of space and time, and has forced us to grapple with the question of the birth and fate of the universe. Yet, despite its subsequently great influence on scientific thought, general relativity was supported initially by very meager observational evidence. It has only been in the last two decades that a technological revolution has brought about a confrontation between general relativity and experiment at unprecedented levels of accuracy. It is not unusual to attain precise measurements within a fraction of a percent (and better) of the minuscule effects predicted by general relativity for the solar system.

To keep pace with these technological advances, gravitation theorists have developed a variety of mathematical tools to analyze the new high-precision results, and to develop new suggestions for future experiments made possible by further technological advances. The same tools are used to compare and contrast general relativity with its many competing theories of gravitation, to classify gravitational theories, and to understand the physical and observable consequences of such theories.

The first such mathematical tool to be thoroughly developed was a "theory of metric theories of gravity" known as the Parametrized Post-Newtonian (PPN) formalism, which was suited ideally to analyzing solar system tests of gravitational theories. In a series of lectures delivered in 1972 at the International School of Physics "Enrico Fermi" (Will, 1974, referred to as TTEG), I gave a detailed exposition of the PPN formalism. However, since 1972, significant progress has been made, on both the experimental and theoretical sides. The PPN formalism has been refined, and new formalisms have been developed to deal with other aspects of

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### Preface to First Edition

xvi

gravity, such as nonmetric theories of gravity, gravitational radiation, and the motion of condensed objects. A recent review article (Will, 1979)<sup>1</sup> summarizes the principal results of these new developments, but gives none of the physical or mathematical details. Since 1972, there has been a need for a complete treatment of techniques for analyzing gravitation theory and experiment.

To fill this need I have designed this study. It analyzes in detail gravitational theories, the theoretical formalisms developed to study them, and the contact between these theories and experiments. I have made no attempt to analyze every theory of gravity or calculate every possible effect; instead I have tried to present systematically the methods for performing such calculations together with relevant examples. I hope such a presentation will make this book useful as a working tool for researchers both in general relativity and in experimental gravitation. It is written at a level suitable for use as either a reference text in a standard graduate-level course on general relativity or, possibly, as a main text in a more specialized course. Not the least of my motivations for writing such a book is the fact that it was my "centennial project" for 1979 – the 100th anniversary of Einstein's birth.

It is a pleasure to thank Bob Wagoner, Martin Walker, Mark Haugan, and Francis Everitt for helpful discussions and critical readings of portions of the manuscript. Ultimate responsibility for errors or omissions rests, of course, with the author. For his constant support and encouragement, I am grateful to Kip Thorne. Victoria LaBrie performed her usual feats of speedy and accurate typing of the manuscript. Thanks also go to Rose Aleman for help with the typing.

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<sup>1</sup>See also Will (1984).