

EXERCISES FOR
FOURIER ANALYSIS

EXERCISES FOR *FOURIER* *ANALYSIS*

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Trinity Hall, Cambridge



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PREFACE

My book *Fourier Analysis* has no exercises and, in my view, is complete without them. However, exercises are useful both to the teacher of a course and to the student who wishes to learn by doing. This supplement provides such exercises (the exercises are grouped by chapter, although not all chapters have exercises).

The two remarks that follow are addressed to students using this book by themselves.

(1) I have tried to produce exercises and not problems. You should find them more in the nature of a hill top walk than a rock climbing expedition. I have marked some of the easier questions with a minus sign to prevent you searching for non-existent subtleties. Very occasionally part of a question is marked by a plus sign to indicate that further reflection may be required.

(2) Unless you intend to do all the questions, you should browse until you find a question or sequence of questions that interest you. You are more likely to pick up knowledge or technique from an exercise that interests you than from one that does not.

The references to other books and papers which occur from time to time are intended to encourage further reading, and not as a complete record of any indebtedness to other sources. The Cambridge Tripos examinations of various years have been the largest single source of questions, but experts will recognise the influence of the texts of Helson, Katznelson, Rogosinski, Dym and McKean and many others. Experts will also recall the verses of Kipling.

When Homer smote his blooming lyre,
He'd heard men sing by land and sea:
And what he thought he might require,
He went and took – the same as me!

The market-girls and fishermen,
The shepherds and the sailors, too,
They heard old songs turn up again,
But kept it quiet – same as you.

I should like to thank Mrs L. Clark for typing these exercises with unfailing cheerfulness and competence. Mr T. Wilkins has kindly worked through them all. I owe him a debt of gratitude not only for uncovering numerous errors but for showing that the exercises can be done. Since it is, in the nature of things, harder to find errors in exercises than in plain text I have no doubt that quite a few remain and I would be grateful to be told of them. My e-mail address is **twk@pmms.cam.ac.uk**.

At the suggestion of my wife I dedicate this book to two charming, though non mathematical, problems – our children Michael and Katy.