Contents

Preface ix
Notation xiii
Introduction 1

I Algebraic Foundations 8
I.1 Fields and Algebras 8
I.2 Integrality and Noetherian properties 26

II Dedekind Domains 35
II.1 Algebraic Theory 35
II.2 Valuations and absolute values 58
II.3 Completions 70
II.4 Module theory over a Dedekind domain 87

III Extensions 102
III.1 Decomposition and ramification 102
III.2 Discriminants and different 120
III.3 Non-ramified and tamely ramified extensions 132
III.4 Ramification in Galois extensions 142

IV Classgroups and Units 152
IV.1 Elementary results 152
IV.2 Lattices in Euclidean space 156
IV.3 Classgroups 164
IV.4 Units 168
Table of Contents

V Fields of low degree
  V.1 Quadratic fields 175
  V.2 Biquadratic fields 193
  V.3 Cubic and sextic fields 198

VI Cyclotomic Fields
  VI.1 Basic theory 205
  VI.2 Characters 213
  VI.3 Quadratic fields revisited 220
  VI.4 Gauss sums 231
  VI.5 Elliptic curves 241

VII Diophantine Equations
  VII.1 Fermat’s last theorem 251
  VII.2 Quadratic forms 254
  VII.3 Cubic equations 269

VIII L-functions
  VIII.1 Dirichlet series 277
  VIII.2 The Dedekind zeta-function 283
  VIII.3 Dirichlet L-functions 295
  VIII.4 Primes in an arithmetic progression 297
  VIII.5 Evaluation of \( L(1, \chi) \) and explicit class number
    formulae for cyclotomic fields 299
  VIII.6 Quadratic fields, yet again 306
  VIII.7 Brauer relations 309

Appendix A. Characters of Finite Abelian Groups 327

Exercises 335

Suggested Further Reading 349

Glossary of Theorems 352

Index 353