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0521438284 - Weak Chaos and Quasi-Regular Patterns

G. M. Zaslavsky, R. Z. Sagdeev, D. A. Usikov and A. A. Chernikov

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This book presents the fundamentals of chaos theory in conservative systems, providing a systematic study of the theory of transitional states of physical systems which lie between deterministic and chaotic behaviour. The authors' treatment of transitions to chaos, the theory of stochastic layers and webs, and the numerous applications of this theory, particularly to pattern symmetry, will make the book of importance to scientists from many disciplines.

The authors begin with an introductory section covering Hamiltonian dynamics and the theory of chaos. Attention is then turned to the theory of stochastic layers and webs and to the applications of the theory. The connection between the various structural properties of the webs and the symmetry properties of patterns is investigated, including discussion of dynamic generators of patterns, hydrodynamic patterns and fluid webs. The final section of the book contains a fascinating collection of patterns in art and living nature. The authors have been meticulous in providing a detailed presentation of the material, enabling the reader to learn the necessary computational methods and to apply them in other problems. The inclusion of a significant amount of computer graphics will also be an important aid to understanding.

The book will be of importance to graduate students and researchers in physics and mathematics who are investigating problems of chaos, irreversibility, statistical mechanics and theories of spatial patterns and symmetries. The perhaps unconventional links between chaos theory and other topics will add to the book's interest.

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***Weak chaos and
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Weak chaos and quasi-regular patterns

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Preface

How does the onset of chaos in Hamiltonian systems occur? This is one of the key questions in the modern theory of dynamic systems. However narrowly specialist this question may seem, the answer has a bearing on almost every branch of physics, including the quantum theory.

Chaos emerges as a result of specific local instability with respect to arbitrarily small perturbations of the system's orbits. It manifests itself in certain regions of phase space and within a certain range of the system's parameters. But the most remarkable feature of chaos is the fact that it is irremovable in fairly general physical situations. What is meant is the following. Under fairly typical conditions in phase space and in the space of values of parameters there always exist such regions in which the dynamics of the system is stochastic. These regions may be arbitrarily small, nevertheless, for a certain structure of the dynamic system given by its Hamiltonian, they are irremovable at any finite values of parameters. An illuminating example of this situation is Arnold's diffusion – a universal, unlimited transport of particles along the channels of a stochastic web in systems with the number of degrees of freedom exceeding two.

As we transfer from systems totally free of stochastic dynamics to systems with chaos, we encounter small regions which are seeds of chaos. In Hamiltonian systems these are stochastic layers and stochastic webs which, being the manifestation of weak chaos in these systems, at the same time perform a certain partitioning of phase space. Thus, as we see, topological properties of phase space turn out to be closely tied to the features and topology of certain regions – the seeds of chaos.

Two of the authors began studying stochastic layers in 1966 in connection with the problem of destruction of magnetic surfaces in toroidal magnetic traps. It was very soon discovered that the result concerning the existence of irremovable stochastic layers is universal. Later, it became clear that a set of stochastic layers can merge in a connected network,

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forming a stochastic web. This stochastic web permeating the entire phase space plays the leading role in the problem of global stability. No wonder that these problems have found numerous applications. Among the most unexpected applications are problems connected with the existence of liquid (hydrodynamic) webs and problems connected with the symmetry of regular and almost regular patterns in condensed matter.

Now it is clear that all these questions are of much interest to a broad audience of scientists. The authors have tried here to present the theory of formation of stochastic layers and webs (the theory of weak chaos), as well as its numerous applications. We have been meticulous in presenting all technical details (which are, in fact, rather simple). This will enable the reader to learn the necessary computational methods and apply them in other problems. The inclusion of a large amount of computer graphics will help to provide a better understanding of the subject.

The book has the following structure. It is divided into four parts. Part I contains necessary information concerning Hamiltonian dynamics and the theory of chaos (for more information, refer to the book by R. Z. Sagdeev, D. A. Usikov and G. M. Zaslavsky: *Nonlinear Physics* (Harwood Academic Publishers, New York, 1988)).

Part II is called 'Dynamic order and chaos'. It considers theories of stochastic layers and webs and their applications.

Part III is called 'Spatial patterns'. As it turned out, various structural properties of the webs are connected with patterns in condensed matter and, among other things, with their symmetry properties. In this part we discuss dynamic generators of patterns, hydrodynamic patterns and liquid webs.

Finally, Part IV, a kind of miscellany, is dedicated to patterns in art and living nature (phyllotaxis).

Generally speaking, the book does not require any technical knowledge and is meant also for students (especially postgraduates) of physics, mathematics and engineering. The authors also hope that those who take a special interest in this field of knowledge and those who have made chaos their speciality will find in this book much interesting and rather unconventional information concerning the theory of chaos: the connection of the theory of chaos to the theory of quasi-crystals, to the theory of plane tilings, to chaotic rotations of a satellite, to errors of computational schemes, etc.

The authors are deeply grateful to their colleagues M. Yu. Zakharov, A. I. Neishtadt, M. Ya. Natenson, B. A. Petrovichev, who participated in obtaining some of the results discussed in the book.