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C. J. S. Clarke
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The theorems of Hawking and Penrose show that space-times are likely to contain incomplete geodesics. Such geodesics are said to end at a singularity if it is impossible to continue the space-time and geodesic without violating the usual topological and smoothness conditions on the space-time. In this book the different possible singularities are defined, and the mathematical methods needed to extend the space-time are described in detail. The results obtained (many appearing here for the first time) show that singularities are associated with a lack of smoothness in the Riemann tensor. While the Friedmann singularity is analysed as an example, the emphasis is on general theorems and techniques rather than on the classification of particular exact solutions.

Cambridge Lecture Notes in Physics 1

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The Analysis of Space-Time Singularities

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The Analysis of Space-Time Singularities

C. J. S. CLARKE
University of Southampton

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Preface

The central aim of this book is the development of the results and techniques needed to determine when it is possible to extend a space-time through an "apparent singularity" (meaning, a boundary-point associated with some sort of incompleteness in the space-time). Having achieved this, we shall obtain a characterisation of a "genuine singularity" as a place where such an extension is not possible. Thus we are proceeding by elimination: rather than embarking on a direct study of genuine singularities, we study extensions in order to rule out all apparent singularities that are not genuine. It will turn out, roughly speaking, that the genuine singularities which then remain are associated either with some sort of topological obstruction to the construction of an extension, or with the unboundedness of the Riemann tensor when its size is measured in a suitable norm.

I had at one stage hoped that there would be a single simple criterion for when such an extension cannot be constructed, which would then lay down once and for all what a genuine singularity is. But it seems that this is not to be had: instead one has a variety of possible tools and concepts for constructing extensions, and when these fail one declares the space-time to be singular on pragmatic grounds. The main such tools are the use of Hölder and Sobolev norms of functions, used for measuring the extent to which the metric or the Riemann tensor is irregular.

The introduction of Sobolev norms, in addition to Hölder ones, is an attempt to carry the work in this book beyond that in my previous publications (see Clarke, 1982). The need for this is twofold: first, the use of this norm enables one to make a connection with the existence theorems for Einstein's equations, which are all formulated in terms of Sobolev spaces. Second, the requirement that a Sobolev norm be definable places a much weaker condition on the Riemann tensor than is the case with Hölder norms or differentiability conditions in the pointwise sense, the Riemann tensor not even having to be bounded in the case of the Sobolev norms relevant to general relativity. Thus the failure of the Sobolev norm at a genuine singularity implies that the Riemann tensor has there a decidedly worse behaviour than is the case with a failure of Hölder continuity.

The penalty paid for this potential improvement is the technical difficulty of linking differential geometry with Sobolev norms on the Riemann tensor, a difficulty compounded by the fact that most relativists are unfamiliar with the techniques of real analysis used in the Sobolev space theory. I shall therefore be devoting some space in the book to explaining these techniques and sketching those key results in the theory that show how the Sobolev spaces enter into existence theorems and singularity theory. Some of these difficulties remain to be overcome, so that it has not proved possible to give a complete treatment using only Sobolev norms on the Riemann tensor. At present the situation appears to be that one can work either with Hölder norms on the Riemann tensor, or Sobolev norms on the metric components. But I have tried to give the Sobolev versions of results wherever possible since it appears that these are ultimately the most significant ones in the context of Einstein's equations.

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