

This textbook is a general introduction to the dynamics of astrophysical fluids for students with knowledge of basic physics at undergraduate level. No previous knowledge of fluid dynamics or astrophysics is required because the author develops all new concepts in context. The first four chapters cover classical fluids, relativistic fluids, photon fluids, and plasma fluids, with many cosmic examples being included. The remaining six chapters deal with astrophysical applications, stars, stellar systems, astrophysical plasmas, cosmological applications, and the large-scale structure of the Universe.

Astrophysical fluid dynamics is a promising branch of astronomy, with wide applicability. This textbook considers the role of plasma and magnetism in planets, stars, galaxies, the interplanetary, interstellar and intergalactic media, as well as the Universe at large.



Astrophysical Fluid Dynamics



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Preface

This small book is intended as a general introduction to astrophysical fluid dynamics. The reader is presumed to possess a knowledge of basic physics, namely, classical physics, elements of relativity, and introductory ideas about quantum mechanics. No previous knowledge of fluid dynamics or of astrophysics is required, these topics being introduced in the book. Although fluid dynamics may constitute a complementary, original, natural, fecund, unexplored, simple, and enjoyable way to introduce astrophysics, the topic of astrophysical fluid dynamics is a promising, distinct, and particularly wide branch of astrophysics at the present time.

The first part of the book (Chapters 1–4) deals with basic fluid dynamics. Although it could also be used for non-astrophysical purposes, it was written with the former in mind. It often includes cosmic examples that are mainly related to a stationary, static, and stratified atmosphere. These conditions provide the greatest simplification while maintaining a high degree of astrophysical interest.

Following the first chapter on classical fluids, Chapter 2 is devoted to relativistic fluids. The early introduction of relativistic fluids is necessary, as many cosmic fluids, and the cosmic fluid itself, are relativistic. One important advantage is that radiative transfer can be developed as transport in a relativistic fluid, thereby avoiding the usual classical mis-interpretation of the radiative Boltzmann equation. Plasmas and magnetohydrodynamics are also included because of their growing interest in the field of astrophysics. The important role played by magnetic fields in a large sample of cosmic systems is only now beginning to be appreciated.

The second part of the book deals with astrophysical applications. The fluid in a star (stellar interiors), the fluid of stars (dynamics of stellar systems), astrophysical plasma fluids (solar magnetic phenomena, interstellar gas dynamics, accretion discs and jets . . .) and three chapters devoted to cosmology and large-scale structure formation (the Newtonian cosmic fluid, the relativistic cosmic fluid, and the fluid of galaxies) are the main topics discussed. The list is reasonably complete. To a greater, or lesser, extent we have considered planets, stars, and galaxies, interplanetary, interstellar, and intergalactic media, as well as the Universe as a whole.

As elementary particle theories and unification models are neither assumed nor introduced here, the history of the fluid that composes our Universe can only be traced back to a time shortly before annihilation. This poses a strict space-time limit on our objectives: $z \approx 10^{10}$.



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The development of each chapter, and even of each section, is obsessively similar, reproducing the same logical scheme: the microscopic Boltzmann equation followed by its 'daughter' equations of macroscopic interest, continuity, motion, and energy balance. Only in those chapters with a relativistic treatment are the last two equations merged. This unified presentation not only has logical and aesthetic grounds, but is also didactic in that it attempts to demonstrate the fruitfulness of a systematic hydrodynamical approach.

E. Battaner January 1995



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