MATHEMATICAL METHODS FOR CAD

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Preface

The content of this book was taught three years in the "D.E.A. d'Analyse Numérique de l'Université Pierre et Marie Curie" (Paris). The book appeared in French in 1990 (Méthodes Mathématiques pour la CAO, RMA 18, Masson). This book is an introduction to CAD from a mathematical and thoretical point of view. The assertions and results are all proved, and the practical considerations (such as implementation, programming language, description of up to date systems of CAD, etc ...) are not studied.

The most important part of the book is the study of "B-splines" functions, which are more and more used these days. Their remarkable mathematical properties are studied in detail, and also the geometric properties of the curves and surfaces one may define with them. "Bézier" curves, historically used before B-splines, are introduced here as special B-spline curves.

Following an idea of C. De Boor and Hölig ([DB-H]), B-splines are defined without the use of divided differences, which lightens the exposition. However, a section recalls the definition and the main properties of divided differences, in order that the reader could be able to refer easily to books adopting this viewpoint. Moreover, the definition of B-splines with divided differences may be generalized in a natural way to the several variables case, which permits the definition of polyhedral splines.

Surfaces are studied from the viewpoints "tensor product" and "triangular Bézier patches". "Polyhedral" spline surfaces (and in particular "box-splines") are also considered, even though they are not yet used in the existing CAD systems.

I have also included a little "algorithmic geometry", which is also a useful domain for CAD. The only question treated here in this regard, which is one of the main tools of the theory, is the problem of automatic triangulation (with a method known under the name of "Voronoï-Delaunay").

In the last part, some algebraic problems are considered, and a brief introduction to the properties of polynomials from the point of view of formal computation is given. This matter is at the moment the subject of much study, and in my opinion will soon become an essential tool for CAD, especially for the resolution of specific problems, such as the intersection of surfaces for instance, when classical methods are deficient. This part is only an introduction to the area, giving the occasion to go over again or to learn some fundamental algebraic techniques (such as Sturm sequences, or resultants) essential for formal computation.

The book is organized as follows.

Chapter 1 introduces non-uniform B-splines (i.e., defined with any knot sequence of **R**) and the basic algorithms used in the theory (evaluation at a point, differentiation algorithm, insertion of a new knot). The "regularizing" property of B-splines is shown, and the chapter ends with a presentation of the "divided differences" point of view.

Chapter 2 is devoted to the study of B-splines and Bézier curves (these are introduced as a particular case of B-spline curves), and to the geometric signifiance of the algorithms given in Chapter 1. Several subdivision algorithms (such as the "Oslo" algorithm for instance) are given, in the general case of B-splines, as well as in the Bézier curve case.

Chapter 3 treats the interpolation problem of a family of points by a spline curve, and of several complements, such as geometric continuity, rational spline or Bézier curves, rational representations of conics, etc ...

Chapter 4 is devolved to surfaces, mainely from the classical point of view of bicubic splines (i.e., tensor products of cubic B-splines), and on the triangular Bézier patches. The rational point of view is also studied, especially the problem and the usefulness of base points. The chapter ends with a rather complete study of polyhedral splines and box-splines.

Chapter 5 presents some tools of algorithmic geometry, especially the problem of triangulation (hopefully automatic) of a domain (not necessarily plane), when the vertices of simplices are fixed. The method of "Voronoï-Delaunay" is explained; this method is known to be optimal (or at least a good one) from the point of view of the shape of the simplices, at least in the case of the plane.

Chapter 6 is the last one and gives an insight into the theory of formal computation, and its relations with geometric problems. In particular, some properties of roots of real polynomials are studied, as well as the resultant of two polynomials, and the notion of real semi-algebraic sets.

I thank the staff of the " Laboratoire d'Analyse Numérique de l'Université Paris 6" for their welcome, and its director Philippe Ciarlet who invited me to teach CAD in the frame of" D.E.A. d'Analyse Numérique".

I thank also Professors Fiorot and Sablonnière, who introduced me to the subject of B-splines in the Rennes Seminar of May 1987 (see [Re]). I used much ofthe material of this seminar in the early chapters of this book. I thank also Michel Merle who gave me his personal notes about polyhedral splines.