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Reflection groups and Coxeter groups

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James E. Humphreys
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In this graduate textbook Professor Humphreys presents a concrete and up-to-date introduction to the theory of Coxeter groups. He assumes that the reader has a good knowledge of algebra, but otherwise the book is self-contained making it suitable either for courses and seminars or for self-study.

The first part is devoted to establishing concrete examples. Chapter 1 develops the most important facts about finite reflection groups and related geometry, leading to the presentation of such groups as Coxeter groups. In Chapter 2 these groups are classified by Coxeter graphs, and actual realizations are described. Chapter 3 discusses in detail the polynomial invariants of finite reflection groups. The first part ends with the construction in Chapter 4 of the affine Weyl groups, a class of Coxeter groups which plays a major role in Lie theory.

The second part (which is logically independent of, but motivated by, the first) starts by developing from scratch the properties of Coxeter groups in general, including the Bruhat ordering. In Chapter 6, it is shown how earlier examples and others fit into the general classification of Coxeter graphs. Chapter 7 introduces the seminal work of Kazhdan and Lusztig on representations of Hecke algebras associated with Coxeter groups. Finally, Chapter 8 sketches a number of interesting complementary topics as well as connections with Lie theory.

The book concludes with an extensive bibliography on Coxeter groups and their applications.

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Preface

'Les choses, en effet, sont pour le moins doubles.'
Proust, *La Fugitive*

Since its appearance in 1968, Bourbaki [1] (treating Coxeter groups, Tits systems, reflection groups, and root systems) has become indispensable to all students of semisimple Lie theory. An enormous amount of information is packed into relatively few pages, including detailed descriptions of the individual root systems and a vast assortment of challenging 'exercises'. My own dog-eared copy (purchased at Dillon's in London in the spring of 1969 for 90 shillings) is always at hand. The present book attempts to be both an introduction to Bourbaki and an updating of the coverage, by inclusion of such topics as Bruhat ordering of Coxeter groups. I was motivated especially by the seminal 1979 paper of D.A. Kazhdan and G. Lusztig [1], which has led to rapid progress in representation theory and which deserves to be regarded as a fundamental chapter in the theory of Coxeter groups.

Part I deals concretely with two of the most important types of Coxeter groups: finite (real) reflection groups and affine Weyl groups. The treatment is fairly traditional, including the classification of associated Coxeter graphs and the detailed study of polynomial invariants of finite reflection groups.

Part II is for the most part logically independent of Part I, but lacks motivation without it. Chapter 5 develops the general theory of Coxeter groups, with emphasis on the 'root system' (following Deodhar [4]), the Strong Exchange Condition of Verma, and the Bruhat ordering. Special cases such as finite and hyperbolic Coxeter groups occupy Chapter 6. Chapter 7 is mainly an exposition of Kazhdan–Lusztig [1]. Finally, Chapter 8 sketches some related topics of interest, with suggestions for further reading. Because the subject reaches out in so many directions, I have provided an extensive (though by no means complete) bibliography.

The arguments in Part I are largely self-contained. However, the treatments of crystallographic reflection groups (Weyl groups) in Chapter 2 and affine Weyl groups in Chapter 4 require some facts about (crystallographic) root systems which are less directly connected with the theory of Coxeter groups and are therefore only summarized here. The coverage in Chapter VI of Bourbaki is thorough and accessible, and highly recommended for the serious student of Lie theory.

There are interesting groups generated by 'reflections' which are not in a natural way Coxeter groups, including for example most of the complex reflection groups (these deserve a book of their own). I have mentioned such related theories only in passing, in order to concentrate the treatment on Coxeter groups.

The history of the subject is long and intricate: see the *Note historique* in Bourbaki as well as the historical remarks in Coxeter [1]. Often a result has been first observed empirically (using the classification of finite reflection groups, for example) and later proved conceptually. I have tried to attribute theorems correctly, but have stopped short of reconstructing the history of each. The notes and references at the ends of chapters are intended to make it possible for the interested reader to get back to the original sources, notably the pioneering work of Coxeter and Witt. I hope readers will call omissions or errors to my attention.

All cross-references are to sections, such as 2.7. Each section contains at most one result labelled lemma, proposition, theorem or corollary, later referred to as (for example) Theorem 2.7. In order to emphasize what I take to be the high points in the development, I have made a distinction (admittedly subjective) between the labels 'proposition' and 'theorem'. Against considerable odds, I have struggled to make consistent notational choices, but there are occasional local aberrations. Exercises are scattered throughout the text. The reader is encouraged to try all of them; but none is required afterwards except as indicated.

I am indebted to the many people whose books, papers, and lectures have shaped my own knowledge of the subject, especially N. Bourbaki, V.V. Deodhar and J. Tits. Special thanks are due to George Avrunin for initiating me into the mysteries of LaTeX. Research support from the National Science Foundation is also gratefully acknowledged.

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For this printing, a number of misprints and minor errors have been corrected, and portions of 4.5, 5.5, 5.10 have been rewritten. I am grateful to the many readers who pointed out errors and suggested improvements, especially J. B. Carrell, E. Neher, and L. Tan.