

Contents

Introduction	1
Part I. The basic theory	
Chapter 1. Invariant measures and some ergodic theory	5
1.1 Invariant measures,	5
1.2 Poincaré recurrence,	9
1.3 Ergodic measures,	9
1.4 Ergodic decomposition,	10
1.5 The ergodic theorem,	12
1.6 Proof of the ergodic theorem,	15
1.7 Proof of the ergodic decomposition lemma,	18
Notes.	19
Chapter 2. Ergodic theory for manifolds and Liapunov exponents	21
2.1 The subadditive ergodic theorem,	21
2.2 The subadditive ergodic theorem and diffeomorphisms,	22
2.3 Oseledec-type theorems,	23
2.4 Some examples,	25
2.5 Proof of the Oseledec theorem,	31
2.6 Further refinements of the Oseledec theorem,	36
2.7 Proof of the subadditive ergodic theorem,	37
Notes.	40
Chapter 3. Entropy	43
3.1 Measure theoretic entropy,	43
3.2 Measure theoretic entropy and Liapunov exponents,	46
3.3 Topological entropy,	48
3.4 Topological entropy and Liapunov exponents,	50
3.5 Equivalent definitions of measure theoretic entropy,	53
3.6 Proof of the Pesin-Ruelle inequality,	58
3.7 Oseledec's theorem, topological entropy and Lie theory,	60
Notes.	62

(viii)

Chapter 4. The Pesin set and its structure	63
4.1 The Pesin set,	64
4.2 The Pesin set and Liapunov exponents,	68
4.3 Liapunov metrics on the Pesin set,	69
4.4 Local distortion,	71
4.5 Proofs of Propositions 4.1 and 4.2,	73
4.6 Liapunov exponents with the same sign,	76
Notes.	77
An interlude	79
(a) Some topical examples,	79
(b) Uniformly hyperbolic diffeomorphisms:	
(i) Shadowing, (ii) Closing lemma, (iii) Stable manifolds,	83
Notes.	85
Part II. The applications	
Chapter 5. Closing lemmas and periodic points	87
5.1 Liapunov neighborhoods,	87
5.2 Shadowing lemma,	90
5.3 Uniqueness of the shadowing point,	94
5.4 Closing lemmas,	95
5.5 An application of the closing lemma,	96
Notes.	98
Chapter 6. Structure of “chaotic” diffeomorphisms	99
6.1 The distribution of periodic points,	99
6.2 The number of periodic points,	101
6.3 Homoclinic points,	103
6.4 Generalized Smale horse-shoes,	105
6.5 Entropy stability,	108
6.6 Entropy, volume growth and Yomdin’s inequality,	110
6.7 Examples of discontinuity of entropy,	115
6.8 Proofs of propositions 6.1 and 6.2,	119
Notes.	122

Cambridge University Press

978-0-521-43593-2 - Lectures on Ergodic Theory and Pesin Theory on Compact Manifolds

Mark Pollicott

Table of Contents

[More information](#)

(ix)

Chapter 7. Stable manifolds and more measure theory	123
7.1 Stable and unstable manifolds,	123
7.2 Equality in the Pesin-Ruelle inequality,	127
7.3 Foliations and absolute continuity,	129
7.4 Ergodic components,	132
7.5 Proof of stable manifold theorem,	133
7.6 Ergodic components and absolute continuity,	137
Notes.	137
Appendix A. Some preliminary measure theory	139
Appendix B. Some preliminary differential geometry	145
Appendix C. Geodesic flows	151
References	155