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978-0-521-43593-2 - Lectures on Ergodic Theory and Pesin Theory on Compact Manifolds

Mark Pollicott

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Centro de Matemática

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