Fourier Integrals in Classical Analysis is an advanced monograph concerned with modern treatments of central problems in harmonic analysis. The main theme of the book is the interplay between ideas used to study the propagation of singularities for the wave equation and their counterparts in classical analysis. Using microlocal analysis, the author, in particular, studies problems involving maximal functions and Riesz means using the so-called half-wave operator.

This self-contained book starts with a rapid review of important topics in Fourier analysis. The author then presents the necessary tools from microlocal analysis, and goes on to give a proof of the sharp Weyl formula which he then modifies to give sharp estimates for the size of eigenfunctions on compact manifolds. Finally, at the end, the tools that have been developed are used to study the regularity properties of Fourier integral operators, culminating in the proof of local smoothing estimates and their applications to singular maximal theorems in two and more dimensions.

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Fourier integrals in classical analysis

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To my family

Contents

Preface

Except for minor modifications, this monograph represents the lecture notes of a course I gave at UCLA during the winter and spring quarters of 1991. My purpose in the course was to present the necessary background material and to show how ideas from the theory of Fourier integral operators can be useful for studying basic topics in classical analysis, such as oscillatory integrals and maximal functions. The link between the theory of Fourier integral operators and classical analysis is of course not new, since one of the early goals of microlocal analysis was to provide variable coefficient versions of the Fourier transform. However, the primary goal of this subject was to develop tools for the study of partial differential equations and, to some extent, only recently have many classical analysts realized its utility in their subject. In these notes I attempted to stress the unity between these two subjects and only presented the material from microlocal analysis which would be needed for the later applications in Fourier analysis. I did not intend for this course to serve as an introduction to microlocal analysis. For this the reader should be referred to the excellent treatises of Hörmander [5], [7] and Treves [1].

In addition to these sources, I also borrowed heavily from Stein [4]. His work represents lecture notes based on a course which he gave at Princeton while I was his graduate student. As the reader can certainly tell, this course influenced me quite a bit and I am happy to acknowledge my indebtedness. My presentation of the overlapping material is very similar to his, except that I chose to present the material in the chapter on oscillatory integrals more geometrically, using the cotangent bundle. This turns out to be useful in dealing with Fourier analysis on manifolds and it also helps to motivate some results concerning Fourier integral operators, in particular the local smoothing estimates at the end of the monograph.

Roughly speaking, the material is organized as follows. The first two chapters present background material on Fourier analysis and stationary phase that will be used throughout. The next chapter deals with non-homogeneous oscillatory integrals. It contains the L^2 restriction theorem for the Fourier transform, estimates for Riesz means in $\mathbb{R}^n$, and Bourgain's circular maximal theorem. The goal of the rest of the monograph is mainly to develop generalizations of these results. The first step in this direction is to present some basic background material from the theory of pseudo-differential operators, emphasizing the role of stationary phase. After the chapter on pseudo-differential operators comes one dealing with the sharp Weyl formula of Hörmander [4], Avakumovič [1], and Levitan [1]. I followed the exposition in Hörmander's paper, except that the Tauberian condition in the proof of the Weyl formula is stated in terms of L^∞ estimates for eigenfunctions. In the next chapter, this slightly different point of view is used in generalizing some of the earlier results from Fourier analysis in $\mathbb{R}^n$ to the setting of compact manifolds. Finally, the last two chapters are concerned with Fourier integral operators. First, some background material is presented and then the mapping properties of Fourier integral operators are investigated. This is all used to prove some recent local smoothing estimates for Fourier integral operators, which in turn imply variable coefficient versions of Stein's spherical maximal theorem and Bourgain's circular maximal theorem.

It is a pleasure to express my gratitude to the many people who helped me in preparing this monograph. First, I would like to thank everyone who attended the course for their helpful comments and suggestions. I am especially indebted to D. Grieser, A. Iosevich, J. Johnsen, and H. Smith who helped me both mathematically and in proofreading. I am also grateful to M. Cassorla and R. Strichartz for their thorough critical reading of earlier versions of the manuscript. Lastly, I would like to thank all of my collaborators for the important role they have played in the development of many of the central ideas in this course. In this regard, I am particularly indebted to A. Seeger and E. M. Stein.

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