

Cambridge University Press
0521432766 - Exercises for Fourier Analysis
T. W. Korner
Frontmatter
[More information](#)

EXERCISES FOR
FOURIER ANALYSIS

EXERCISES FOR
FOURIER
ANALYSIS

T. W. KÖRNER

Trinity Hall, Cambridge



CAMBRIDGE
UNIVERSITY PRESS

Cambridge University Press
0521432766 - Exercises for Fourier Analysis
T. W. Korner
Frontmatter
[More information](#)

Published by the Press Syndicate of the University of Cambridge
The Pitt Building, Trumpington Street, Cambridge CB2 1RP
40 West 20th Street, New York, NY 10011-4211, USA
10 Stamford Road, Oakleigh, Melbourne 3166, Australia

© Cambridge University Press 1993

First published 1993
Reprinted 1996

A catalogue record for this book is available from the British Library

Library of Congress cataloguing in publication data

Körner, T. W. (Thomas William), 1946–
Exercises for Fourier analysis / T. W. Körner.
p. cm.
ISBN 0-521-43276-6 (hardback). – ISBN 0-521-43849-7 (pbk.)
1. Fourier analysis—Problems, exercises, etc. I. Körner, T. W.
(Thomas William), 1946– Fourier analysis. II. Title.
QA403.5.K66 1993
515'.723—dc20 92-30757 CIP

ISBN 0 521 43276 6 hardback
ISBN 0 521 43849 7 paperback

Transferred to digital printing 2002

CONTENTS

<i>Preface</i>	ix
1 Introduction	1
2 Proof of Fejér's theorem	9
3 Weyl's equidistribution theorem	11
4 The Weierstrass polynomial approximation theorem	18
5 A second proof of Weierstrass's theorem	25
6 Hausdorff's moment problem	30
7 The importance of linearity	32
8 Compass and tides	34
9 The simplest convergence theorem	39
10 The rate of convergence	41
11 A nowhere differentiable function	46
12 Reactions	49
13 Monte Carlo methods	51
15 Pointwise convergence	53
16 Behaviour at points of discontinuity I	55
17 Behaviour at points of discontinuity II	59
18 A Fourier series divergent at a point	63
19 Pointwise convergence, the answer	82
20 The undisturbed damped oscillator does not explode	85
21 The disturbed damped linear oscillator does not explode	95
23 The linear damped oscillator with periodic input	97
27 Poisson summation	100
28 Dirichlet's problem	106
29 Potential theory with smoothness assumptions	107
30 An example of Hadamard	111
31 Potential theory without smoothness assumptions	114
32 Mean square approximation I	122
33 Mean square approximation II	133

34	Mean square convergence	135
35	The isoperimetric problem I	140
36	The isoperimetric problem II	141
37	The Sturm–Liouville equation I	142
38	Liouville	144
39	The Sturm–Liouville equation II	157
40	Orthogonal polynomials	164
41	Gaussian quadrature	168
43	Tchebychev and uniform approximation I	171
44	The existence of the best approximation	178
45	Tchebychev and uniform approximation II	179
46	Introduction to Fourier transforms	190
47	Change in the order of integration I	191
48	Change in the order of integration II	192
49	Fejér’s theorem for Fourier transforms	195
50	Sums of independent random variables	197
51	Convolution	199
52	Convolution on \mathbb{T}	200
53	Differentiation under the integral	206
54	Lord Kelvin	211
55	The heat equation	212
57	The age of the earth II	216
59	Weierstrass’s proof of Weierstrass’s theorem	219
60	The inversion formula	220
63	A second approach	244
64	The wave equation	245
71	The central limit theorem II	246
72	Stability and control	247
73	Instability	248
74	The Laplace transform	255
75	Deeper properties	262
76	Poles and stability	284
77	A simple time delay equation	293
79	Many dimensions	298
80	Sums of random vectors	306
81	A chi squared test	307
82	Haldane on fraud	310
86	Will a random walk return?	311
87	Will a Brownian motion return?	313
89	Will a Brownian motion tangle?	314
94	Why do we compute?	315
95	The diameter of stars	321
97	Fourier analysis on the roots of unity	326

<i>Contents</i>		vii
99	How fast can we multiply?	331
102	A good code?	335
103	A little more group theory	336
104	Fourier analysis on finite Abelian groups	345
105	A formula of Euler	347
107	Primes in some arithmetical progressions	349
108	Extension from real to complex variable	351
109	Primes in general arithmetical progressions	358
 <i>Appendixes A, B, G</i>		 366
<i>References</i>		378
<i>Index</i>		381

PREFACE

My book *Fourier Analysis* has no exercises and, in my view, is complete without them. However, exercises are useful both to the teacher of a course and to the student who wishes to learn by doing. This supplement provides such exercises (the exercises are grouped by chapter, although not all chapters have exercises).

The two remarks that follow are addressed to students using this book by themselves.

(1) I have tried to produce exercises and not problems. You should find them more in the nature of a hill top walk than a rock climbing expedition. I have marked some of the easier questions with a minus sign to prevent you searching for non-existent subtleties. Very occasionally part of a question is marked by a plus sign to indicate that further reflection may be required.

(2) Unless you intend to do all the questions, you should browse until you find a question or sequence of questions that interest you. You are more likely to pick up knowledge or technique from an exercise that interests you than from one that does not.

The references to other books and papers which occur from time to time are intended to encourage further reading, and not as a complete record of any indebtedness to other sources. The Cambridge Tripos examinations of various years have been the largest single source of questions, but experts will recognise the influence of the texts of Helson, Katznelson, Rogosinski, Dym and McKean and many others. Experts will also recall the verses of Kipling.

When Homer smote his blooming lyre,
He'd heard men sing by land and sea:
And what he thought he might require,
He went and took – the same as me!

The market-girls and fishermen,
The shepherds and the sailors, too,
They heard old songs turn up again,
But kept it quiet – same as you.

I should like to thank Mrs L. Clark for typing these exercises with unfailing cheerfulness and competence. Mr T. Wilkins has kindly worked through them all. I owe him a debt of gratitude not only for uncovering numerous errors but for showing that the exercises can be done. Since it is, in the nature of things, harder to find errors in exercises than in plain text I have no doubt that quite a few remain and I would be grateful to be told of them. My e-mail address is **twk@pmms.cam.ac.uk**.

At the suggestion of my wife I dedicate this book to two charming, though non mathematical, problems – our children Michael and Katy.