

Index

- ABEL_PQUOTE, 518–20, 523, 535, 545
 ABEL_QUOTE, 521, 523, 540, 545, 557
 abelian group, 9, 517–24
 free, 320
 abelianized relations, 518, 520
 accessible
 automation, 122
 part, 122
 state, 120, 300
 subset construction, 124
 ACTIVE, 189–92, 215, 284, 577–9
 ACTIVE_X, 285–6
 ADJUST, 544, 555–6
 ADJUST_MU, 367–8, 375
 admissible sequence, 407
 AG-system, 394
 aligned monomials, 499–502
 almost shortest vector, 365
 alphabet, 97
 alternating group, 17
 APL language, 571
 ascending chain condition, 17
 associated exponent matrix, 406
 ATTACH, 226
 automatic groups, 5
 automaton, 96, 101, 572–4
 accessible, 122
 compatible, 218
 complete, 105
 coset, 171–5
 deterministic, 105, 297
 finite, 101–2, 108
 generalized, 101, 297
 generalized coset, 301
 important-coset, 163
 index, 112–20, 147
 isomorphic, 104
 minimal, 126
 numeric, 132
 ordinary, 297
 reduced coset, 174
 Schreier, 102
 extended, 269
 standard, 130–41
 trim, 120
 automorphism, 16
 inner, 16
 group, 16
 Axiom computer system, 5, 570

 BACK_TRACE, 181–2, 215, 223, 284, 287
 BACK_TRACE_X, 287–9
 backtrack search, 35–40, 116, 140, 204, 252
 backward cone, 100
 BALANCE, 84–7
 basic
 sequence, 437–44
 wreath product ordering, 48, 137–8, 193,
 258–60, 398, 437
 BASIC language, 571
 BASIC_CE, 185–6, 192, 195–8, 210, 220,
 223
 basis of an abelian group, 320
 Baumslag, Gilbert, 448, 556
 Bayer, David, 2
 binary
 operation, 8
 search, 83
 bit string, 10
 Boone, William, 41–2
 Buchberger, Bruno, 448
 BUILD, 225–6
 Burnside group, 561

 C language, 29, 31, 571–3
 CA_JOIN, 202

- Cannon, John J., 5
 Cannonito, Frank B., 448, 556
 CANON, 161, 210
 canonical form, 51, 88, 161
 Cayley, Arthur, 40
 Cayley
 automaton, 102
 computer system, 5, 564, 570–1
 diagram, 102
 center, 16
 centralizer, 417–19
 Chinese remainder theorem, 34
 Church-Rosser property, 54–5
 circuit, 123
 class
 exponent- p , 446
 nilpotency, 386
 CLASS_2_PQUOTE, 537–8, 540, 545, 547
 CLASS_2_QUOTE, 541–2, 544–5, 551, 555, 557
 classical niladic rewriting system, 153, 217
 closed set, 111
 closure
 normal, 16
 of a set of states, 111
 coaccessible state, 120, 300
 coefficient, 455
 COINCIDENCE, 179–92, 215, 223–5, 228, 231–4, 239, 253, 284
 coincidence procedure, 179–92, 310
 COINCIDENCE_X, 286, 288–9
 COINCSP, 578–9
 collected word, 395
 collection, 66, 401, 405, 436
 combinatorial, 404
 from the left, 401, 403
 from the right, 401–2
 to the left, 66, 401
 column operation, 333
 modulo d , 346
 combinatorial collection, 404
 commutation relation, 398
 commutator(s), 245, 384, 562–3
 basic sequence of, 437–44
 left-normed, 384
 subgroup, 384
 compatible automaton, 218
 complete automaton, 105
 completion, 110
 complexity theory, 2
 composition, 7
 COMPRESS, 576
 computational
 algebra, 2
 group theory, 1–5
 computer algebra, 1
 concatenation, 103
 cone, 97–100
 backward, 100
 trim, 98
 confluence, 54–5, 57–65
 local, 54–5, 424
 CONFLUENT, 62–3, 65, 69, 116
 confluent presentation, 57
 congruence
 class, 34
 of integers, 34
 on an automaton, 178
 on a monoid, 18–19
 right, 88–95
 congruent integers, 34
 conjugacy
 algorithms, 405
 classes, 16
 problem, 28, 417–19
 CONJUGATE, 199–201
 conjugate elements, 16
 consequence, 24
 consistency, 420, 430–5
 consistent
 polycyclic presentation, 396
 with length, 45
 with multiplication, 488–9
 consonant module elements, 487
 constant term, 510
 construction
 derivation, 157, 213
 subset, 111–12
 accessible, 124
 with automata, 141–7, 210
 contraction, 299
 as coset automaton, 306
 corner entries, 322
 coset, 15
 automaton, 171–5, 573, 575
 cyclically reduced, 200
 reduced, 174, 213
 enumeration, 96, 150, 175–92, 210, 214, 223, 266, 571
 extended, 276–90, 295
 generalized, 277
 modified, 277
 restartable, 264
 important, 162–71
 table, 171

- standard, 203–10
- critical pair, 489, 504
- cycle, 17
- cyclic
 - decomposition, 308
 - extension, 419–30
 - group, 12
 - HLT strategy, 245
 - monoid, 12
 - permutation, 10
- cyclically reduced coset automaton, 200
- decision procedure, 218
- decomposition, cyclic, 308
- deduction, 176, 185
 - stack, 176, 234, 237, 245
- DEFINE, 176, 178, 182–4, 187, 199, 214, 223, 225–8, 233–4, 236–9, 241, 246, 284
- DEFINE_X, 284, 288
- defining
 - pair, 163
 - relation, 20
- definition, 400, 430
- degree, 510
 - of a word, 49
- derivable from, 54
 - in one step, 52
- derivation construction, 157, 213
- derived
 - length, 386
 - series, 385
 - subgroup, 384
- deterministic automaton, 105, 297
- discriminant, 364
- edge, 101, 297
 - standard, 133–5
- effective
 - degree, 449, 457, 467
 - total degree, 473, 478
- elementary
 - abelian p -group, 445, 517
 - matrix, 330
 - operation, 406, 410
- empty word, 10
- endomorphism, 12
- endpoint, 103
- Epstein, David B. A., 5
- equivalence, 406
 - modulo d , 346
 - relation, 7
- equivalent
 - edges, 191
 - matrices, 333
- Euclidean algorithm, 32
 - extended, 33
- even permutation, 17
- expanding morphism, 126, 168
- expansion, 298, 304
- exponent- p class, 446
- exponent of a group, 15
- EXTEND_RULES, 532–4, 541, 543, 551–2, 557–8
- extended
 - coset enumeration, 276–90, 295
 - Euclidean algorithm, 33
 - Schreier automaton, 269
 - transition table, 269
- extension
 - cyclic, 419–30
 - formal, 524–34
 - split, 526
- FELSCH, 235, 239–41
- Felsch strategy, 232–9, 245, 266, 580
- finite automaton, 101–2, 108
- finitely generated
 - group, 11
 - monoid, 11
- finitely presented
 - group, 23
 - monoid, 20
- FIRST_CONJ, 208–10, 253
- FORCE, 202–3
- formal extension, 524–34
- FORTRAN language, 571
- Fratini subgroup, 446
- freely reduced word, 22
- free
 - abelian group, 320
 - equivalence, 22
 - group, 22
 - monoid, 13
 - nilpotent group, 401, 436–45
 - product, 64
 - semigroup, 13
- freely reduced word, 22
- full sequence, 409
- fundamental theorem of finitely generated
 - abelian groups, 337
- G_S, 361–2
- Galois, Evariste, 40
- γ -weighted presentation, 400–1, 430–1, 447
- GAP computer system, 5, 570–2

- GCD, 32–3
 GCDX, 33–5, 350, 357
 general linear group, 9
 GENERAL_CE, 219–20, 222–3
 generalized
 automaton, 101, 297
 coset automaton, 301
 coset enumeration, 277
 word problem, 28
 generating set
 for a group, 11
 for a monoid, 11
 for a monoid ideal, 14
 minimal, 14–15
 GENERATORS, 262–3, 266
 Gram-Schmidt procedure, 361
 greatest common divisor, 32
 GRÖBNER, 491–4, 507
 Gröbner basis, 35, 448, 482–94, 556–7, 571, 573
 algorithm, 512–13
 reduced, 493
 GRÖBNER_L, 505–7
 group, 9, 15–18
 abelian, 9, 517–24
 alternating, 17
 cyclic, 12
 dihedral, 159
 exponent of, 15
 finitely generated, 11
 finitely presented, 23
 free, 22
 abelian, 320
 nilpotent, 401, 436–45
 general linear, 9
 hopfian, 393–4
 metabelian, 386
 nilpotent, 386–90, 417–19
 of units, 11
 permutation, 17
 polycyclic, 319, 383, 390, 392
 polycyclic-by-finite, 383
 presentation, 23
 quotient, 16
 ring, 448, 507–12, 556
 solvable, 386–90
 symmetric, 9
 with automatic structure, 96
 GRP_REDUCE, 485–9, 491, 504

 Hadamard bound, 344–5, 348–9, 356
 Havas, George, 5, 239, 580
 HEAD, 49–50

 head, 49
 Hermite, Charles, 382
 Hermite normal form, *see* row Hermite normal form
 Higman, Graham, 562
 Hilbert basis theorem, 449, 490
 Hirsch number, 394
 HLT, 228, 231, 237, 239–41, 284
 HLT strategy, 227–32, 245, 266, 277, 283–90, 296–7, 308, 318, 580
 cyclic, 245
 HLT_X, 284, 288–9, 523
 Holt, Derek, 5
 homomorphism, 12
 hopfian group, 393
 Howson's theorem, 197
 hybrid tables, 314

 ideal
 in a monoid, 13–15, 99
 right, 99
 of polynomials, 449–55
 identity element, 8
 important cosets, 162–71
 important-coset automaton, 163
 INDEX, 118, 120
 index, 15
 automaton, 83, 112–20, 147
 INDEX_CONFLUENT, 117–19
 INDEX_REWRITE, 113–14, 119
 initial state, 101
 inner
 automorphism, 16
 product, 360
 invariant factor, 338, 518
 modulo d , 346
 inverse
 element, 8
 relation, 7
 irreducible
 module element, 485
 word, 52
 isomorphism, 12
 theorems, 16

 JOIN, 177–8, 182, 185, 187, 214, 216, 223, 233–4, 236–8, 253, 284
 JOIN_X, 285, 288
 joining, 310

 Kannan-Bachem algorithm, 350, 358
 KB_COL, 357–8, 360
 KB_DIAG, 358–9

- KB_ROW, 350–1, 354–8, 360, 378–9, 381
- KBS_1, 70–1, 75–6, 81–3
- KBS_2, 76–9, 81–3, 85, 89–90, 93, 149–50, 265
- kernel, 16
- Knuth, Donald, 3, 39
- Knuth-Bendix procedure, 5, 68–96, 119, 147–50, 210–11, 263–6, 251, 291–3, 329, 396, 399, 417, 436–7, 494, 512–13, 568, 571
- label, 101, 297
 - primary, 269
 - secondary, 269
- language, 96–100
 - rational, 96, 98–100, 108–9, 155–7
- recognized by automaton, 103
- lattice, 363–5
- Laurent
 - monomial, 495
 - polynomial, 448, 495–507, 509
- leading
 - coefficient, 472, 484, 502
 - exponent, 395
 - generator, 484
 - monomial, 472, 502
 - term, 395, 472, 484, 500–1
- left end, 49
- left-normed commutator, 384
- length
 - function, 116–17
 - of a vector, 361
- length-plus-lexicographic ordering, 45
- LENLEX_STND, 135–6, 195
- lexicographic ordering, 44–5
- linear ordering, 43
- LISP, 571
- LLL, 367–71, 376
- LLL-algorithm, 5, 365, 367, 571
- LLL-reduced basis, 365, 381
- local confluence, 54–5, 424
- lookahead, 231–2
- LOW_INDEX, 254, 256–8, 260
- low-index subgroup algorithm, 252
- lower
 - central series, 385, 417
 - exponent- p central series, 445
- Lysenok, I. G., 561
- Macaulay computer system, 2, 5, 570–2
- MACSYMA computer system, 5, 570
- Maple computer system, 367, 379–81, 570–1
- Mathematica computer system, 379–81, 570–1
- Mathieu group, 293
- matrices, equivalent, 333
- matrix
 - elementary, 330
 - transition, 136
 - unimodular, 330
- MEMBER, 327, 350
- MERGE, 190–2, 215, 284, 578–9
- MERGE_X, 285–6, 289
- METABEL_QUOT, 558, 560
- metabelian group, 386, 495
- Miller, Charles F., III, 448, 556
- minimal decomposition, 213
- MLLL, 374, 376, 379–81
- Möbius function, 440
- modified
 - coset enumeration, 277
 - LLL algorithm, 365, 372–8
- module, 448
 - presentation, 460
- monadic rewriting system, 575
- monoid, 8–15, 18–22
 - congruence on, 18
 - cyclic, 12
 - finitely generated, 11
 - finitely presented, 20
 - free, 13
 - ideal of, 13–15
 - presentation, 20
 - quotient, 19
- morphism, expanding, 126, 168
- MPROD, 31
- Myhill-Nerode theorem, 96
- natural map, 7
- Neubüser, Joachim, 4–5, 266
- Newman, Maxwell, 4
- Newman, Michael F., 5, 303, 406
- NEXT_PQUOT, 546–7, 550
- NEXT_QUOT, 550, 552, 555–6
- Nielsen reduced set, 93–5
- Nielsen reduction, 93, 95, 210
- niladic rewriting system, 151–9
 - classical, 153, 217
 - general, 212–16
- nilpotency class, 386
- nilpotent
 - group, 386–90, 417–19
 - presentation, 399, 401
 - quotient algorithm, 514–15, 517
- normal
 - closure, 16
 - subgroup, 16

- NORMAL_GENS, 263, 292
 normalizer, 200
 Novikov, Petr, 41
 numeric automaton, 132
- order of an element, 12
 ordering
 - by component, 456
 - by degree, 456
 - consistent with length, 94
 - length-plus-lexicographic, 45
 - lexicographic, 44–5
 - linear, 43
 - reduction, 46
 - well-, 43
 - wreath product, 46–50
 - basic, 48, 137–8, 193, 258–60, 398, 437
- ordinary automaton, 297
 orthogonal vectors, 361
 overlap, 59
 - proper, 59
- OVERLAP_1, 69–72, 77
 OVERLAP_2, 77, 79, 265
- p*-group, 445–7, 564
 - elementary abelian, 445, 517
- p*-quotient algorithm, 514–15, 517, 567
 Pari computer system, 5, 570–2
 part
 - accessible, 122
 - trim, 121, 300
- partition
 - of a set, 7
 - of an integer, 36
- Pascal language, 29, 31, 32, 571, 573
 path, 103, 297
 - compression, 188–9
- permutation group, 17
 perpendicular vectors, 361
 φ -weighted presentation, 447, 564
 Pohst, Michael, 365
 POLY_MEMBER, 407, 411, 414
 POLY_SUBGROUP, 412–13, 415
 polycyclic
 - generating sequence, 394, 397–8, 414, 509
 - group, 319, 383, 390, 392
 - presentation, 398
 - consistent, 396
 - standard, 395, 397, 399, 417, 436
 - standard monoid, 395, 421
 - quotient algorithm, 448, 514–15, 517
 - series, 390
- polycyclic-by-finite group, 383
 polynomial, 510
 positive word, 23
 Post, Emil, 41
 power relation, 398
 power-commutator presentation, 400
 power-conjugate presentation, 399, 404
 precedence, 132
 prefix, 10
 - confluent, 89
 - reduced, 89–90
- presentation
 - confluent, 57
 - γ -weighted, 400–1, 430–1, 447
 - group, 23
 - monoid, 20
 - multiplication table, 63
 - nilpotent, 399, 401
 - of a module, 460
 - φ -weighted, 447, 564
 - polycyclic, 398
 - power-commutator, 400
 - power-conjugate, 399, 404
 - restricted, 65, 528
 - semigroup, 21
 - simplification, 263, 290–4
 - weakly γ -weighted, 405
- primary
 - label, 269
 - signature, 269
- PRINT, 574–5
 PROBE, 38
 PROCESS_DEDUCTIONS, 234–7, 239, 246
 product, free, 64
 proper overlap, 59
- queue, 33
 quotient
 - coset automaton, 179
 - group, 16
 - monoid, 19
- quotients of derived series, 388
- rank
 - of a free abelian group, 320
 - of a free group, 22
 - of a matrix, 327
 - modulo d , 343
- rational language, 96, 98–100, 108–9, 155–7
 recognition of a language, 103
 REDUCE computer system, 5
 REDUCE (for coset automata), 174–6, 198, 200, 214

- REDUCE (for module elements), 489, 491, 493–4, 504, 506, 570
- REDUCE_L, 505–6
- reduced
 - coset automaton, 174, 213
 - Gröbner basis, 493
 - rewriting system, 56
 - word, 52
- reduction
 - of integer matrices, 35
 - ordering, 46, 52
- regular homomorphism, 515
- Reidemeister-Schreier procedure, 268–95, 571
- relation, 7
 - defining, 20
 - equivalence, 7
 - \mathbb{Z} -, 372
- relatively prime integers, 34
- resolution procedure, 512–13
- restricted
 - Burnside group, 561
 - presentation, 65, 528
- restriction, 110
- reversal, 10
- reverse lexicographic ordering, 472, 492
- REWRITE, 52–3, 55–7, 59, 62, 185, 200, 285–7, 486
- REWRITE_FROM_LEFT, 67–70, 76–7, 82, 85, 114
- rewriting, 52, 66–8, 83, 147
 - from the left, 67
 - rule, 51
 - system, 51, 53
 - monadic, 575
 - niladic, 151–9
 - see also* collection
- right
 - congruence, 88–95
 - ideal in a monoid, 99
- ROD, 350–3, 356–7
- Rosser strategy, 326, 379
- row
 - echelon form, 322
 - equivalence, 321, 330
 - modulo d , 342
 - Hermite normal form, 322–3, 328–9, 331, 347–50, 354, 356, 360, 365, 497
 - modulo d , 342
 - operation, 321
 - modulo d , 342
 - reduction, 323
- ROW_REDUCE, 323, 326, 331–2, 354, 378
- RS_BASIC, 272–4, 283
- RS_SGEN, 272–5
- RS_SGEN2, 275–6
- rule
 - rewriting, 51
 - identifier, 113–14
- S -polynomial, 487
- SAC2 computer system, 5, 570–2
- Schreier
 - automaton, 102
 - extended, 269
 - diagram, 102
 - generator, 165, 263, 290, 292, 294
- Schreier-Todd-Coxeter method, 263
- SEARCH, 36, 40
- search tree, 36–9
- secondary
 - label, 269
 - signature, 269
- semicritical pair, 487, 504
- semigroup, 8
 - free, 13
 - presentation, 21
- signature, 103, 297
 - primary, 269
 - secondary, 269
- simplification of presentations, 263, 290–4
- small-gap strategy, 239
- Smith, Henry J. S., 382
- Smith normal form, 333–7, 339, 346–7, 349–50, 358–9, 365, 520
 - modulo d , 346
- solvable group, 386–90
- special
 - columns, 577–8
 - decomposition, 36–40
- split extension, 526
- splitting, 309
- stack, 32
 - deduction, 176, 234, 237, 245
- STANDARD, 195–6
- standard
 - automaton, 130–41
 - coset tables, 203–10
 - form, 406, 411–12, 484
 - monoid polycyclic presentation, 395, 421
 - polycyclic presentation, 395, 397, 399, 417, 436
 - polycyclic rewriting system, 396
- standardization, 132, 135, 138, 161, 186, 192–6, 245

- STANDARDIZE, 220, 228, 235, 240, 247, 289
 standardizing strategies, 239–45
 starting point, 103
 state, 101
 accessible, 120, 300
 coaccessible, 120, 300
 initial, 101
 intermediate, 103
 terminal, 101
 trim, 120, 300
 Stillman, Michael, 2
 subgroup, 10
 membership problem, 28
 normal, 16
 submonoid, 10
 subset construction, 111–12
 accessible, 124
 subword, 10
 suffix, 10
 susceptible to cancellation, 213
 SWITCH, 193, 195–6, 240, 247
 switch, 192–6
 Sylow's theorem, 41
 symbolic computation, 1–2
 SYMM, 502–3, 506
 symmetric group, 9
 symmetrized set, 502
 system, rewriting, 51, 53
 syzygy, 460–1

 table(s)
 coset, 171
 extended transition, 269
 hybrid, 314
 transition, 106
 TEN_CE, 245–8, 250–1, 265
 terminal state, 101
 test element, 487
 TEST_1, 69–72
 TEST_2, 76–8, 86–7
 Tietze transformation, 25, 263, 291, 537–8, 542, 551
 Todd-Coxeter procedure, 175, 182, 264, 266
 total degree, 472
 effective, 473, 478
 ordering, 472
 TRACE, 105–7, 169, 181–2, 200, 223, 284, 287
 trace record, 311
 TRACE_X, 287–9
 transition
 matrix, 136
 table, 106
 extended, 269
 translation invariance, 45–6
 TRIM, 121–3, 143
 trim
 automaton, 120
 cone, 98
 part, 121, 300
 state, 120, 300
 Turing, Alan, 4
 Turing machine, 41
 TWO_SIDED_TRACE, 182–3, 185, 187, 215, 223, 228, 232–6, 238, 246, 284, 288
 TWO_SIDED_TRACE_X, 288–9

 unimodular matrix, 330
 UNION, 188–90
 union-find procedure, 187, 211
 unit, 8–9

 verification, 27, 218, 568

 weakly γ -weighted presentation, 405
 weight, 50, 437, 564
 well-ordering, 43
 word, 10
 collected, 395
 positive, 23
 problem, 26–8, 460
 generalized, 28
 wreath product ordering, 46–50, 258, 266
 WREATH_STND, 138, 210

 Z-relation, 372
 ZBASIS, 463–4, 471
 ZBASIS_TD, 480–1
 Zel'manov, E. I., 561
 ZXIDEAL, 452–5, 461–2, 464, 466, 483, 496
 ZXIDEALREP, 454–5, 464, 466, 471
 ZXMODULE, 459–60, 466, 468–71
 ZXSERIES, 464–6, 468–9, 471
 ZXYIDEAL_TD, 480–2
 ZXYMEMBER, 473
 ZXYMODULE, 468–9, 471