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Absolutely Summing Operators

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Introduction

The roots of the theory of p-summing operators lie in work undertaken by Alexandre Grothendieck from the 1950s. However, it was only in 1967 that Albrecht Pietsch clearly isolated this class of operators and established many of their fundamental properties. Within a year Pietsch's work gained recognition thanks to the appearance of the seminal paper "Absolutely summing operators in $\mathcal{L}_p$-spaces and their applications" of Joram Lindenstrauss and Aleksander Pełczyński.

The time was ripe for these ideas to gain ascendance. They soon made serious inroads into other areas of analysis – witness Pełczyński's influential monograph "Banach Spaces of Analytic Functions and Absolutely Summing Operators" which was published within a decade of Pietsch's introduction – and this trend has continued unabated.

Unfortunately, students and specialists in subjects other than Banach space theory have scant sources to learn about our topic. P. Wojtaszczyk's excellent and broad ranging "Banach Spaces for Analysts" is pitched at a level high enough to be uncomfortable for many students. Indeed, with the possible exception of G.J.O. Jameson's elegant text "Summing and Nuclear Norms in Banach Space Theory", which carefully avoids the measure theoretical underpinnings of the subject, no general introduction to the area exists. We hope our book goes some way towards filling this gap.

Our aim is to introduce the student of analysis to the basic facts about p-summing operators and their relatives and to show how their theory can be applied in a variety of situations. We have tried to follow the dictum espoused by C.A. Rogers:

"As the book is largely based on lectures, and as I like my students to follow my lectures, proofs are given in detail; this may bore the mature mathematician, but it will I believe be a great help to anyone trying to learn the subject *ab initio*."

We expect that those with a sound background in real analysis, complex analysis and functional analysis will be ready and able to gain from the study of this text. As indicated above, we have presented much of the material in classes and seminars, and this has greatly influenced our ordering.

The first six chapters give a thorough and, we hope, accessible introduction which should be suitable for presentation in classes, seminars and the like. Naturally, the material in later chapters is often much tougher, and sometimes we find it convenient to call on substantial results from other areas without proof. On occasion, theorems are given two different proofs; as one might expect, this is to illustrate how various developments lend new insights.

Despite the length of the text, we have striven to avoid being encyclopaedic. However, at the end of each chapter, we have notes and remarks in which we relate historical information, indicate directions to be pursued, provide results that are a bit beyond our perceived scope, and generally pander to our encyclopaedic urges without poisoning the text. We make no claim of completeness (although we always work with Banach spaces). Hopefully, we have been accurate in allocating priorities.

As is usual in such ventures, we have benefitted from the support of many people and institutions. We take the opportunity to thank some of those to whom we are indebted.

The Mathematics Departments at Kent State University and Universität Zürich have provided us with stages where we tried our material on the most valuable of audiences – graduate students and colleagues. To all these people we extend our thanks for their patience and perceptive advice.

We have also had the opportunity to test our wares at a number of other institutions, most particularly, Centrale (Caracas, Venezuela), Complutense (Madrid, Spain), Los Andes (Merida, Venezuela), New England (Armidale, Australia), PAN (Warsaw, Poland), Pretoria (Republic of South Africa), São Paulo (Brazil), Sevilla (Spain) and University College (Dublin, Ireland).

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NOTATION

We shall stick to standard definitions and terminology. $\mathbf{N}$, $\mathbf{Q}$, $\mathbf{R}$, $\mathbf{C}$ will be the systems of natural, rational, real and complex numbers; $\mathbf{Z}$ will denote the ring of integers, and occasionally we shall write $\mathbf{N}_0 := \mathbf{N} \cup \{0\}$. We shall also use $\mathbf{K}$ to stand for $\mathbf{R}$ or $\mathbf{C}$.

Our main interest is in Banach spaces over $\mathbf{K}$ and in their operators. Throughout this text, X, Y, Z, ... will be Banach spaces; typical members will be $x, y, z, \ldots$, respectively, perhaps with indices. When we deal with finite dimensional spaces, E, F, H, ... may be used to denote the space in question. By span M and conv M we mean the linear span and the convex hull of a subset M of a Banach space.

The norm of a Banach (or just normed) space X will usually be denoted by $\|\cdot\|$, but when more precision is desirable, we may also use $\|\cdot\|_X$. B_X will denote the closed unit ball $\{x \in X : \|x\| \leq 1\}$ of our Banach space X, whereas $S_X = \{x \in X : \|x\| = 1\}$ is its unit sphere.

As usual, bounded linear maps between Banach spaces are referred to as operators; they will be denoted by u, v, w, ... Let X and Y be Banach spaces. The collection $\mathcal{L}(X, Y)$ of all operators $u : X \to Y$ is a Banach space with respect to the (uniform) norm $\|u\| = \sup_{x \in B_X} \|ux\|$. If $X = Y$, then $\mathcal{L}(X)$ is used instead of $\mathcal{L}(X, X)$. We shall extend this convention to subsystems of operators as well. For example, $\mathcal{F}(X, Y)$ will be used to denote the collection of all operators in $\mathcal{L}(X, Y)$ which have a finite dimensional range, and $\mathcal{F}(X)$ replaces $\mathcal{F}(X, X)$.

A Banach space operator $u : X \to Y$ is *isometric* if $\|ux\| = \|x\|$ for all $x \in X$; it is an *isometry* if it is in addition onto.

The continuous dual of a Banach space X is $X^* := \mathcal{L}(X, \mathbf{K})$; its typical member will be denoted by x^*, and for $x \in X$, we shall write $\langle x^*, x \rangle$ (or $\langle x, x^* \rangle$) for the action of x^* on x. The bidual of X is the space $X^{**} = (X^*)^*$, and $x \mapsto \langle \cdot, x \rangle$ establishes an isometric embedding $X \hookrightarrow X^{**}$; we denote it by k_X.

Only in Chapter 18 do we deviate from our notation concerning elements; we do this to avoid potential confusion with the notation of $*$-algebras.

Hilbert spaces will generally be denoted by H or H_k, and if x and y are elements of a Hilbert space their inner product will be written $(x|y)$.

The classical function spaces $C(K)$ of continuous $\mathbf{K}$-valued functions on a compact Hausdorff space K as well as the spaces $L_p(\mu)$ relative to a measure μ need no further comment. The latter are Banach spaces when $1 \leq p \leq \infty$.

Occasionally, we shall also take into account the case $0 < p < 1$; then we are dealing with complete metrizable topological vector spaces. If μ is the counting measure on some given set J, we write ℓ_p^J in place of $L_p(\mu)$. Thus ℓ_∞^J just consists of all bounded functions $f : J \to \mathbf{K}$, and $\|f\|_\infty = \sup_{j\in J}|f(j)|$. If p is finite, then $f : J \to \mathbf{K}$ belongs to ℓ_p^J if and only if $\{j \in J : f(j) \neq 0\}$ is countable and $\sum_{j\in J}|f(j)|^p$ is finite; this sum is then $\|f\|_p^p$.

By c_0^J we mean the closed subspace of ℓ_∞^J consisting of all functions $f : J \to \mathbf{K}$ which 'vanish at infinity': given $\varepsilon > 0$ there is a finite subset M of J such that $f(j) \leq \varepsilon$ for all $j \in J \setminus M$.

The classical sequence spaces ℓ_p and c_0 are obtained by taking $J = \mathbf{N}$. If J is a finite set, $J = \{1,\dots,n\}$ say, then we prefer to write ℓ_p^n in place of ℓ_p^J.

At certain important junctures we shall also encounter the classical Lorentz spaces $L_{p,q}(\mu)$ relative to a measure space (Ω, Σ, μ); we shall only be interested in the case $1 \leq p, q \leq \infty$. $L_{p,q}(\mu)$ consists of all (μ-a.e. equivalence classes of) measurable functions $f : \Omega \to \mathbf{K}$ whose *decreasing rearrangement*

$$f^* : [0, \mu(\Omega)) \longrightarrow \mathbf{K} : t \mapsto \inf\{s > 0 : \mu(|f| > s) \leq t\}$$

satisfies

$$\|f\|_{p,q} := \Big(\int_0^{\mu(\Omega)} \big(t^{(1/p)-1} f^*(t)\big)^q dt\Big)^{1/q} < \infty$$

if $q < \infty$, and

$$\|f\|_{p,\infty} := \sup_{t\geq 0} t^{1/p}|f^*(t)| < \infty$$

if $q = \infty$. The spaces obtained in this way are Banach spaces though $\|\cdot\|_{p,q}$ is, in general, only equivalent to a Banach space norm. But we always have $L_{p,p}(\mu) = L_p(\mu)$ isometrically. Details can be found in many books, for example in E.M. Stein and G. Weiss [1971].

If μ is counting measure on $\mathbf{N}$ (or $\{1,\dots,n\}$) we write $\ell_{p,q}$ (or $\ell_{p,q}^n$) instead of $L_{p,q}(\mu)$. When we take the absolute values of the members of a scalar sequence (a_k) and arrange them in non-increasing order, we obtain the decreasing rearrangement (a_k^*), and then

$$\|(a_k)\|_{p,q} = \Big(\sum_n \big(n^{(1/p)-1} a_n^*\big)^q\Big)^{1/q}$$

if $q < \infty$, whereas

$$\|(a_k)\|_{p,\infty} = \sup_n n^{1/p} \cdot a_n^*.$$

Let $1 \leq p < \infty$. If $(X_j)_{j\in J}$ is a family of Banach spaces, then its ℓ_p *direct sum*,

$$\big(\textstyle\bigoplus_{j\in J} X_j\big)_p,$$

(also written $\big(\bigoplus_{j\in J} X_j\big)_{\ell_p^J}$) consists of all $(x_j)_{j\in J} \in \prod_{j\in J} X_j$ such that $(\|x_j\|)_{j\in J}$ belongs to ℓ_p^J. This is a Banach space with respect to the norm

$$\|(x_j)_{j\in J}\| = \|(\|x_j\|)_{j\in J}\|_p = \big(\textstyle\sum_{j\in J}\|x_j\|^p\big)^{1/p}.$$

The meaning of $\left(\bigoplus_{j\in J} X_j\right)_\infty$ and $\left(\bigoplus_{j\in J} X_j\right)_{c_0}$ is self-explanatory. As in the scalar case, we make the canonical identification of the dual of $\left(\bigoplus_{j\in J} X_j\right)_{c_0}$ with $\left(\bigoplus_{j\in J} X_j^*\right)_1$; the duality is given by $\langle (x_j^*), (x_j)\rangle = \sum_{j\in J}\langle x_j^*, x_j\rangle$. In the same way, for $1 \le p < \infty$, we shall identify the dual of $\left(\bigoplus_{j\in J} X_j\right)_p$ with $\left(\bigoplus_{j\in J} X_j^*\right)_{p^*}$. The ℓ_p direct sum of n Banach spaces $X_1,\ldots,X_n$ will also be denoted by $X_1 \oplus_p \ldots \oplus_p X_n$.