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978-0-521-42997-9 - Aspects of Combinatorics: A Wide-Ranging Introduction

Victor Bryant

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A WIDE-RANGING INTRODUCTION

Aspects of combinatorics

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Introduction

By its formal dictionary definition ‘combinatorics’ is that branch of mathematics which deals with combinations, and those combinations can be of sets, graphs, matrices, traffic routes, people, etc. etc. Indeed, such an elementary theme is bound to have many applications and to cut across many other branches of mathematics, and that is one of the fascinations of the subject. It is also bound to have many different aspects, some of which are presented here.

Over a decade ago, when courses on this subject were first entering the undergraduate curriculum, combinatorics was often looked upon disparagingly as an easy option. But the popularity of such courses amongst undergraduates is due largely to the fact that at least the *questions* make sense, even though the *answers* are as difficult as in any other course. Another advantage to the student is that the wide range of topics means that even if one chapter is a complete haze the next one offers a fresh start. The advantage to the lecturer of this wide selection of ideas is that he or she can include many miscellaneous topics discovered when reading the latest books and journals.

Of course there are dangers that such a *pot-pourri* will become just a rag-bag of trivia, but as you collect together material for a course you begin to see how deep and inter-related some of the ideas are. Some of these inter-relationships only become clear after teaching the material for many years: there is real pleasure in marking a piece of work where a student is the first to notice that the ‘doctor’s waiting-room’ example from chapter 1 can be used in solving the ‘hostess problem’ in chapter 12, and in discovering that a theorem from tournaments can be proved much more neatly by a simple application of a marriage theorem derived earlier.

My own selection of *Aspects* . . . is, of course, a very personal one and it includes a large amount of graph theory and transversal theory. The subjects are not grouped together but visited again and again throughout the text: I have found that this is the most entertaining way to present the material and that a pause before seeing another application of some theorem enables greater understanding of it. Although there are no formal pre-requisites for studying this material, a lot of it requires a certain mathematical maturity and is unlikely to be suitable for students before at least their second year in higher education. There is more than enough material here for a full year’s course and the lecturer may choose to save some of the topics for a more advanced course or merely to omit some of the harder results and/or proofs.

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Introduction

I would like to place on record my thanks to Hazel Perfect who (together with the late Leon Mirsky) wrote some pioneering papers in transversal theory and who kindly and caringly involved me in their world when I was a newly qualified lecturer. My thanks also go to all the authors whose books and papers have coloured my choice of topics and the way in which I present them. I am also grateful to Peter Brooksbank, a Sheffield graduate now doing his own research, for reading the typescript and making some valuable comments. But most of all my wholehearted thanks go to the many other Sheffield undergraduates and postgraduates who, over the years, have attended my courses in combinatorics and who have shared my enthusiasm for the subject. Teaching them has been a very great pleasure and privilege.

Victor Bryant
1992