Manifold Mirrors

The Crossing Paths of the Arts and Mathematics

Most works of art, whether illustrative, musical or literary, are created subject to a set of constraints. In many (but not all) cases, these constraints have a mathematical nature; for example, the geometric transformations governing the canons of J. S. Bach, the various projection systems used in classical painting, the catalogue of symmetries found in Islamic art or the rules concerning poetic structure. This fascinating book describes geometric frameworks underlying this constraint-based creation. The author provides both a development in geometry and a description of how these frameworks fit the creative process within several art practices. Furthermore, he discusses the perceptual effects derived from the presence of particular geometric characteristics.

The book began life as a liberal arts course and is certainly suitable as a textbook. However, anyone interested in the power and ubiquity of mathematics will enjoy this revealing insight into the relationship between mathematics and the arts.

Felipe Cucker is Chair Professor of Mathematics at the City University of Hong Kong. His research covers a variety of subjects, including semi-algebraic geometry, computer algebra, complexity, emergence in decentralized systems (in particular, emergence of languages and flocking), learning theory and foundational aspects of numerical analysis. He serves on the editorial board of several journals and is Managing Editor of the journal *Foundations of Computational Mathematics*, published by the Society of the same name.

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FELIPE CUCKER
City University of Hong Kong

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MATHEMATICS: USER'S MANUAL

Launch not beyond your depth, but be discreet,
And mark that point where sense and dullness meet.

A. Pope (1966: An Essay on Criticism)

The writer of any book dealing with mathematics who wishes to reach a broad audience invariably faces a dilemma: How to describe the mathematics involved. No matter how well motivated the intervening notions, nor how lengthily described, the question that eventually will pose itself is what to do regarding proofs.

Working mathematicians are generally reluctant to dispense with them, and I am no exception. For, on the one hand, a proof of a statement shows its necessity, its truth with respect to an underlying collection of assumptions. And, on the other hand, in doing so, it usually conveys an intuition on the nature of the objects occurring in the statement. This intuition is of the essence. It decreases the confusion that the alternation of definitions and statements in the mathematical discourse naturally creates.

Occasionally, however, the understanding afforded by a proof does not compensate for the effort of its reading. This may be so because one already has a form of the intuition mentioned above (and would, therefore, feel annoyed by having to "prove the evident") or because the proof is too involved and fails to convey any intuition. In these cases the task of following the proof's details becomes boring.

In this trade-off between boredom and confusion[1] different readers find different solutions by choosing subsets of proofs to be read that best suit their circumstances. To make these choices possible, this book gives proofs for many of its (mathematical) statements. To further make it easier, some observations are now given.

The mathematical development of this book is, essentially, self-contained and relies on knowledge widely taught in secondary school courses. In this sense, any person having benefited from these courses will be able to read what follows. Mathematical content, while present throughout the book, is concentrated in Chapters 2, 3, 7, 9, 12 and 13. The first three of these chapters are, essentially, self-contained, in the sense that almost all results therein are proved. For the last three, in contrast, we could not proceed in a like manner without unduly increasing the proportion of mathematics in our exposition.

[1] Another form of this trade-off will be central to the arguments in this book.

Mathematical statements do not share the same conceptual importance. Common practice in mathematical writing describes as "theorems" those results whose statements are goals in themselves. Stepping stones toward the proof of a theorem are called either "Propositions" (when the statement is nevertheless of independent interest) or "Lemmas" (when the statement is understood as subordinate and of a technical nature only [2]). It is left to each individual reader to decide what degree of attention to pay to the mathematical details in the book. An extreme choice would be to read every mathematical result with its proof, provided such a proof is given. The other extreme would be – of course paying attention to the formal definitions and general description of the notions at hand – to skip or skim everything except for the theorems' statements. (In this case, for instance, Sections 2.5, 2.6, 3.3 and 7.4, as well as § 3.5.2 and § 9.2.3, would reduce to a single, simple statement.) An intermediate strategy would be to proceed with an initial reading following the latter choice and then return to the skipped details if the need arises in later chapters. In the choice of this degree of attention the reader is encouraged to keep in mind Pope's advice and fix it at that personal point "where sense and dullness meet".

[2] Sometimes, however, an author writes as a lemma a result which time proves to be of crucial importance. There are numerous examples of lemmas that eventually become worthy of the status of Theorem.