

INDEX OF NOTATION

 $\|\bullet\|$, norm, 3 \mathbb{R}^+ , non-negative reals, 3 \mathbb{C} , complex numbers, 3 \mathbb{R}_n , real n -space, 3 \mathbb{C}_n , complex n -space, 4 $\|\bullet\|_1$, ℓ_1 -norm, 4 $\|\bullet\|_\infty$, sup norm, 4 $C[a, b]$, continuous function on $[a, b]$, 4 (\bullet, \bullet) , inner-product, 5 ℓ_2 , space of square summable sequences, 7 $x \perp y$, orthogonal, 11 ω , positive integers, 12 Σ , finite subsets of ω , 14 $\ell_2(\Gamma)$, square summable functions on Γ , 15 \bar{A} , closure, 16 $\text{sp}S$, linear span of a set S , 18 $(C[a, b], \|\bullet\|_2)$ continuous functions with $\|f\|^2 = \int_a^b |f(t)|^2 dt$, 21 $T: X \rightarrow Y$, function T from X to Y , 25 $\mathcal{L}(H)$, bounded linear operators on Hilbert space, 28 $B(H)$, bounded linear operators on Hilbert space, 28 $\mathcal{R}(A)$, range of the operator A , 28 $M \perp N$, orthogonal subspaces, 30 M^\perp , orthogonal complement, 30 $M \oplus N$, direct sum, 30 Y^* , conjugate space, 31 $\ker f$, kernel of the linear functional f , 31 $\text{dist}(x, K)$, distance from a vector to a set, 33 $\sigma(T)$, spectrum of T , 36 $\rho(T)$, resolvent of T , 36 $\det A$, determinant, 36

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- R_ζ , resolvent of an operator, 37
 C_r , circle of radius r , 39
 r_T , spectral radius, 44
 $\sigma_c(T)$, continuous spectrum of the operator T , 47
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Cambridge University Press

978-0-521-42933-7 - Hilbert Space: Compact Operators and the Trace Theorem

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