

INDEX OF NOTATION

- $\|\bullet\|$, norm, 3
- \mathbb{R}^+ , non-negative reals, 3
- \mathbb{C} , complex numbers, 3
- \mathbb{R}_n , real n -space, 3
- \mathbb{C}_n , complex n -space, 4
- $\|\bullet\|_1$, ℓ_1 -norm, 4
- $\|\bullet\|_\infty$, sup norm, 4
- $C[a, b]$, continuous function on $[a, b]$, 4
- (\bullet, \bullet) , inner-product, 5
- ℓ_2 , space of square summable sequences, 7
- $x \perp y$, orthogonal, 11
- ω , positive integers, 12
- \sum , finite subsets of ω , 14
- $\ell_2(\Gamma)$, square summable functions on Γ , 15
- \bar{A} , closure, 16
- $\text{sp}S$, linear span of a set S , 18
- $(C[a, b], \|\bullet\|_2)$ continuous functions with $\|f\|^2 = \int_a^b |f(t)|^2 dt$, 21
- $T: X \rightarrow Y$, function T from X to Y , 25
- $\mathfrak{L}(H)$, bounded linear operators on Hilbert space, 28
- $B(H)$, bounded linear operators on Hilbert space, 28
- $\mathcal{R}(A)$, range of the operator A , 28
- $M \perp N$, orthogonal subspaces, 30
- M^\perp , orthogonal complement, 30
- $M \oplus N$, direct sum, 30
- Y^* , conjugate space, 31
- $\ker f$, kernel of the linear functional f , 31
- $\text{dist}(x, K)$, distance from a vector to a set, 33
- $\sigma(T)$, spectrum of T , 36
- $\rho(T)$, resolvent of T , 36
- $\det A$, determinant, 36

128 *Index of Notation* R_ζ , resolvent of an operator, 37 C_r , circle of radius r , 39 r_T , spectral radius, 44 $\sigma_c(T)$, continuous spectrum of the operator T , 47 $\sigma_r(T)$ residual spectrum of the operator of T , 47 $\sigma_p(T)$, point spectrum of the operator T , 47 $N(T)$, numerical range of the operator T , 51 $M(T) = \sup_{\|x\|=1} (Tx, x)$, 51 $M(T) = \inf_{\|x\|=1} (Tx, x)$, 51 $\phi(x, y)$, bilinear form, 54 $\psi(x) = \phi(x, x)$, quadratic form associated with the bilinear form ϕ , 54 $K(H)$, compact operators on Hilbert Space, 64 $T^{1/2}$, square root of the operator T , 70 $\sigma_n(T)$, singular numbers of the operator T , 75 ℓ_p , $1 \leq p \leq \infty$, space of p -summable sequences, 78 c_0 , space of sequences converging to zero, 78 $\mathcal{L}(H, \ell_2)$, bounded linear operators from H to ℓ_2 , 81 $\mathcal{S}_p(H)$, $1 \leq p \leq +\infty$, Schatten p -classes, 82 $s_p(T)$, p -class norm, 84 $HS(H)$ Hilbert-Schmidt operators on H , 93 $hs(H)$, Hilbert-Schmidt norm, 94 $\mathcal{L}(X, Y)$, bounded linear operators from X to Y , 94 $\text{tr}A$, trace of A , 95 $TC(H)$, trace class operators, 97 $\nu(T)$, nuclear norm, 97 $\text{tr}(A, B)$, trace of \mathcal{S}_2 -operators A and B , 99 $\phi - \text{tr}(T)$, functional trace of the operator T , 99 $\sigma - \text{tr}(T)$, spectral trace of the operator T , 100 $\Pi_2(X, Y)$, absolutely 2-summing operators from X to Y , 102 $\pi_2(T)$, absolutely 2-summing norm, 104 $N(X)$, nuclear operators, 104

$\alpha_n(T)$, approximation numbers of the operator T , 106

$\ell_1(X)$, operators of type $-\ell_1$, 106

$\tilde{\text{tr}}$, extension of the spectral trace, 110

$\beta_{n,p}(T)$, nth p -mean of $\{\sigma_i(T)\}$, 113

INDEX OF TERMS

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 at most countable, 12
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