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Hilbert Space: Compact Operators and the Trace Theorem

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More Information

TO:

Charles McArthur - Mathematics

Pat - Matrimony

Gunnar Johansen - Music*

*Gunnar Johansen, one of the great romantic pianists, and a friend for 35 years, died May 25, 1991 at the age of 85. Life is diminished by his passing.

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Ron Retherford
Baton Rouge, LA

CONTENTS

INTRODUCTION

These chapters contain the material of a summer course (8 weeks) given at LSU a few years ago and repeated at Johannes Kepler Universität, Linz. In the summer the mathematics department at LSU is faced with offering courses that may be taken by graduate students at *all* levels: beginning to advanced Ph.D.! I hope that this material meets (and fills) that need.

For these lectures, the student will need a bit of mathematical sophistication and a fairly good course in advanced calculus (Cauchy Sequences, convergence of sequences, uniform continuity) and a good course in (finite dimensional) linear algebra (determinants, eigenvalues, linear transformations).

An undergraduate course in complex variables would also be nice. But, if the student was introduced to the line integral in calculus, the complex integration we do in these notes should present no difficulties. Knowledge of Lebesgue measure is *not* assumed. Thus, these notes will not discuss, e.g. $L_2[0,1]$ and thus also will not discuss integral operators given by L_2-kernels. (*To the student*: Forget this paragraph if it fails to make sense.)

Many will say that this omits too much from the theory of compact operators on Hilbert space. I claim not. It omits many important *examples* but in these notes we are interested in the *representation* of compact operators. From this point of view we have omitted nothing!

Our goal is to prove the Lidskij trace formula in as easy a fashion as possible. Another goal is to introduce the student to some Hilbert (and Banach) space techniques. After mastering these notes and a course in measure theory, the student should not have too much trouble absorbing some of the finer points of the general theory. Indeed, these notes have been written so that (with appropriate modifications, definitions, and a little effort) the main results can be generalized to certain classes of operators on general Banach spaces.

There are many exercises scattered through the text, as well as some "why" etc., in the proofs. The student is expected to do *all* the exercises and to try to understand *all* the proofs (answer the "why"!). There are a dozen important inequalities given throughout the text. To prove these inequalities, one often needs lesser inequalities. These easier inequalities have been assigned as exercises. Almost without exception these easier results all are "LaGrange Multiplier problems" and the student should refresh his memory on this subject (maximizing subject to a constraint). Credit, as best I know it, has been given for theorems and proofs.

At the end of each chapter, is a section entitled "Notes, exercises and hints." While the exercises in these sections are not necessary to achieve the proof of the Lidskij trace theorem, they do afford the opportunity for a better understanding of operators on Hilbert spaces and a chance to use some Banach space techniques. Extensive hints are given to keep the entire work user friendly. For those students desiring a challenge, cover up the hints.

These lectures have been influenced by many people, but primarily by H. König and A. Pietsch. I intended to write these lectures for publication a long time ago. I learned that both König and Pietsch were writing books on the subject. I, therefore, decided to wait and see what these books had to offer. These books are now in print (see Future Readings) - they have (as I suspected) much to offer! However, they are both written for the expert and not the novice. Thus I feel, even in this company, that these notes will still serve a useful function.

I chose to keep the references for this work at a minimum. I dropped a lot of names throughout the text. The books of Dunford and Schwartz (I and II) and Pietsch (Eigenvalues and s-numbers) contains hundreds of references concerning the material in these chapters. I recommend then to you (you won't find all the names!). The book of Pietsch also contains an amusing and detailed account of the (generalizations) of the material at hand.

Hopefully, the student will be intrigued by this survey of the the subject and will, at some point, study the books of König and Pietsch, and proceed to research.