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Papers dedicated to A. M. Macbeath

Proceedings of a conference at Birmingham University

Edited by
W. J. Harvey
King's College London

in collaboration with
C. Maclachlan
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Preface

This book is a collection of articles addressing a range of topics in the theory of discrete groups; for the most part the papers represent talks delivered at a conference held at the University of Birmingham in January 1991 to mark the retirement of A. M. Macbeath from his chair at the University of Pittsburgh.

The central theme of the volume is the study of groups from a geometric point of view. Of course, the geometric aspect takes many forms; thus one may find a group which operates on Euclidean or hyperbolic space rubbing shoulders with free groups or some generalisation studied with the help of graphs or homological algebra. Groups with presentation also relate to abstract or formal geometry: in recent years the study of groups which act on trees has brought algebraic structures back into the geometric fold, and the seminal idea that a countable group itself carries a geometric essence has reinforced this return to geometry within rather than through group theory, giving Klein's Programme a fresh cutting edge.

A major part of group theory today relates directly to explicit algebraic or geometric objects — permutation groups, Coxeter groups and discrete subgroups of Lie groups are prominent — and one of the strengths of this field lies in the wealth of fascinating interactions with complex analysis and low dimensional topology. The serious study of discrete groups via combinatorial techniques began in hyperbolic space with Poincaré and Dehn, and the reader will find here many echoes of their original ideas and interests. In particular, applications of the structural properties of Fuchsian and crystallographic groups, the representation of finite groups as automorphisms of surfaces, orientable or not, and the continuing drive to understand the classification of manifolds in low dimension by way of numerical invariants, such as Euler characteristic or invariant volume; for instance, a topic of great interest recently has been the attempt to estimate the minimum volume attained by a compact hyperbolic 3-manifold.

Other articles address wider issues including the arithmetic and analysis of automorphic forms, the geometry of moduli spaces, deformation theory of discrete groups in hyperbolic space and applications in mathematical physics. Several authors provide a review of their general area of study. All reflect something of the field in which Murray Macbeath has worked

Preface

predominantly during much of his mathematical career and this conference provided clear testimony to the continuing vitality of both.

The editors are very grateful to all who helped in the production of this volume, especially to the referees who performed their traditional act of altruism with skill, dedication and (above all) willingness. For the running of the Birmingham conference we are greatly indebted to the London Mathematical Society, which provided a generous grant, to the University of Birmingham Mathematics department for the welcome and hospitality they extended to us, and especially to Dr. A. H. M. Hoare who organised all the local arrangements. Finally, of course, we thank Murray for making it possible both by furnishing the mathematical focus over the years and by providing us with the perfect excuse for such an enjoyable occasion. We all wish him a long, happy and productive retirement.