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Lectures on Mechanics

Jerrold E. Marsden
Department of Mathematics and EECS, University of California at Berkeley

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Preface

Many of the greatest mathematicians — Euler, Gauss, Lagrange, Riemann, Poincaré, Hilbert, Birkhoff, Atiyah, Arnold, Smale — were well versed in mechanics and many of the greatest advances in mathematics use ideas from mechanics in a fundamental way. Why is it no longer taught as a basic subject to mathematicians? *Anonymous*

I venture to hope that my lectures may interest engineers, physicists, and astronomers as well as mathematicians. If one may accuse mathematicians as a class of ignoring the mathematical problems of the modern physics and astronomy, one may, with no less justice perhaps, accuse physicists and astronomers of ignoring departments of the pure mathematics which have reached a high degree of development and are fitted to render valuable service to physics and astronomy. It is the great need of the present in mathematical science that the pure science and those departments of physical science in which it finds its most important applications should again be brought into the intimate association which proved so fruitful in the work of Lagrange and Gauss. *Felix Klein, 1896*

These lectures cover a selection of topics from recent developments in the geometric approach to mechanics and its applications. In particular, we emphasize methods based on symmetry, especially the action of Lie groups, both continuous and discrete, and their associated Noether conserved quantities veiwed in the geometric context of momentum maps. In this setting, *relative equilibria*, the analogue of fixed points for systems without symmetry are especially interesting. In general, relative equilibria are dynamic orbits that are also group orbits. For the rotation group $SO(3)$, these are uniformly rotating states or, in other words, *dynamical motions in steady rotation*.

Some of the main points to be treated are as follows:

- The stability of relative equilibria analyzed using the method of separation of internal and rotational modes, also referred to as the block diagonalization or normal form technique.

- Geometric phases, including the phases of Berry and Hannay, are studied using the technique of reduction and reconstruction.
- Mechanical integrators, such as numerical schemes that exactly preserve the symplectic structure, energy, or the momentum map.
- Stabilization and control using methods especially adapted to mechanical systems.
- Bifurcation of relative equilibria in mechanical systems, dealing with the appearance of new relative equilibria and their symmetry breaking as parameters are varied, and with the development of complex (chaotic) dynamical motions.

A unifying theme for many of these aspects is provided by reduction theory and the associated mechanical connection for mechanical systems with symmetry. When one does reduction, one sets the corresponding conserved quantity (the momentum map) equal to a constant, and quotients by the subgroup of the symmetry group that leaves this set invariant. One arrives at the reduced symplectic manifold that itself is often a bundle that carrying a connection. This connection is induced by a basic ingredient in the theory, the *mechanical connection* on configuration space. This point of view is sometimes called the gauge theory approach to mechanics. The geometry of reduction and the mechanical connection is an important ingredient in the decomposition into internal and rotational modes in the block diagonalization method, a powerful method for analyzing the stability and bifurcation of relative equilibria. The holonomy of the connection on the reduction bundle gives geometric phases. When stability of a relative equilibrium is lost, one can get bifurcation, ***solution symmetry breaking***, instability and chaos. The notion of ***system symmetry breaking*** in which not the solutions but the equations loose symmetry, is also important but here is treated only by means of some simple examples.

Two related topics that are discussed are control and mechanical integrators. One would like to be able to control the geometric phases with the aim of, for example, controlling the attitude of a rigid body with internal rotors. With mechanical integrators one is interested in designing numerical integrators that exactly preserve the conserved momentum (say angular momentum) and either the energy or symplectic structure, for the purpose of accurate long time integration of mechanical systems. Such integrators are becoming popular methods as their performance gets tested in specific applications. We include a chapter on this topic that is meant to be a basic introduction to the theory, but not the practice of these algorithms.

This work proceeds at a reasonably advanced level but has the corresponding advantage of a shorter length. For a more detailed exposition of many of these topics suitable for beginning students in the subject, see my book with Tudor Ratiu, *Mechanics and Symmetry* (Springer-Verlag, 1992).

The work of many of my colleagues from around the world is drawn upon in these lectures and is hereby gratefully acknowledged. In this regard, I especially thank Vladimir Arnold, Judy Arms, John Ball, Tony Bloch, David Chillingworth, Richard Cushman, Michael Dellnitz, Arthur Fischer, Mark Gotay, Marty Golubitsky, John Harnad, Darryl Holm, Phil Holmes, John Guckenheimer, Jacques Hurtubise, Vivien Kirk, P.S. Krishnaprasad, Debbie Lewis, Robert Littlejohn, Ian Melbourne, Vincent Moncrief, Richard Montgomery, George Patrick, Tom Posbergh, Tudor Ratiu, Alexi Reyman, Gloria Sanchez de Alvarez, Shankar Sastry, Jürgen Scheurle, Juan Simo, Ian Stewart, Greg Walsh, Steve Wan, Alan Weinstein, Shmuel Weissman, Steve Wiggins, and Brett Zombro. The work of others is cited at appropriate points in the text.

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