Political economy: A personal interpretation and an overview

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1 Political economy: A personal interpretation

It is very natural to think of dynamical systems as *structurally stable,* where this means that small changes in the initial conditions or in the laws of motion will have no qualitative effect on the process. Of course not all processes can be structurally stable, since it is very easy to construct a process that is unstable (in terms of qualitative transformation after perturbation). Until 1966 it was conjectured that structurally stable systems were *generic*. Here generic is a technical term meaning typical in a strong sense. Roughly speaking, a property of a class of models is generic if (i) the property is true of a dense collection of the models (with respect to an appropriate topology on the models), and (ii) any model with the property when perturbed to a small enough degree still has the property. Peixoto (1962) had generically classified all dynamical systems in two dimensions, showing they could be described by sinks, sources, orbits, and so on, thus demonstrating that structurally stable systems were generic in two dimensions. However Smale (1966) constructed a dynamical system with the feature that small changes lead to an infinite variety of qualitatively different systems. At the heart of this construction was the *chaotic strange attractor,* versions of which have been found in simulations of complex systems in meteorology (the Lorenz attractor) and in many other fields (Stewart 1989). Smale's result showed that structurally stable systems could not, in general, be generic.[1]

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[1] Notice that the Peixoto-Smale result demonstrated that three dimensions were necessary to produce chaos for a dynamical (smooth) system. However, it was later shown (Li and

For social scientists it is equally natural, perhaps, to view the world as essentially deterministic or at least, in principle, predictable. In economic theory, for example, there is a powerful vision of constructing a general equilibrium model of human behavior. It would be arrogant to presume that such a model could even incorporate all aspects of human economic behavior, but it might be reasonable to suppose that it captures the essence of economic production and trade. However, if the true model of human interaction were chaotic, then it would appear very difficult indeed to justify its predictions about human behavior.

The mathematical results on dynamical systems just mentioned show that perfectly reasonable dynamical systems may either be structurally stable (and based on global sinks, sources, limit cycles, etc.) or chaotic (based on strange attractors with fractal dimensions).[2] A priori, there is no reason to suppose that human behavioral systems happen to be of one kind or another. However it is now possible to devise complex statistical tests to check whether chaos exists in real economic or other aggregate data. Recent analyses have conclusively demonstrated the existence of chaos in such data (see Barnett and Chen 1988; Barnett, Gallant, Hinich, and Jensen 1992; Barnett and Hinich 1992) and have estimated the fractal dimension of the strange attractor.

A possible explanation for the occurrence of chaos in economic data is the demonstration by Saari (1991) that a nonlinear difference equation $p_{t+1} = f(p_t)$ (which governs the price changes implied by tâtonnement) can be chaotic. Saari's result depends, in turn, on the existence of an exchange economy whose aggregate excess demand function satisfies the appropriate nonlinear properties.

It is one thing to show the theoretical possibility of chaos in a tâtonnement process, but a much deeper problem, it seems to me, is to model the pattern of human interaction that generates the dynamic process. If such a model could be shown to exhibit a globally stable attractor then we could argue that the process was structurally stable, and make inferences with respect to its predictability. On the other hand, if the properties of the process appear to be deeply susceptible to, say, the structure of beliefs and expectations of the agents being modeled, then chaos may be a real possibility.

Yorke 1976) that chaos can be produced with a 1-dimensional variable x if the change of state rule is discrete, i.e., given by a nonlinear difference equation $x_{t+1} = f(x_t)$. See, for example, May (1975), Rand (1978), and the discussion in Schofield (1980).

[2] It should be mentioned that a dynamical system can be both structurally unstable and nonchaotic. A perturbation of such a system can result in systems of qualitatively different types, either structurally stable or chaotic. Typically, systems of this kind are nowhere dense, and lie on the boundary of classes of chaotic or structurally stable systems.

Arrow's theorem (1951) was extraordinarily prescient in this respect. The focus of his analysis, as I understand it, was the construction of a social welfare function incorporating information concerning all individual preferences. Arrow permitted arbitrary preferences on a finite set of alternatives and showed that the requirement of strong rationality in the social welfare function, together with the usual Pareto (or unanimity) principle, effectively implied dictatorship.

The proof of Arrow's result depended on the demonstration that, for each nondictatorial and Paretian aggregation method, there exists at least one preference profile which gives a social preference violating the rationality rule. However, one may rephrase the Arrow problem for the differentiable category in the following way. Suppose the set of alternatives W has a differentiable structure, and suppose further that f is a simple (or first-order) aggregation procedure that maps a profile $u=(u_1,\dots,u_n)$ of preferences or utilities (in a space U), and a list $s=(s_1,\dots,s_n)$ of individual characteristics (such as resources) in a space S, into a "social" behavioral rule $f(u,s)$, namely a state transition function or correspondence, on the space W. The strong rationality principle considered by Arrow can be interpreted as the requirement that f be a gradient process, defined in terms of, say, a maximizing principle. If this process exhibits a globally stable attractor, then the process cannot be chaotic. Arrow's result suggests that when f is a gradient system then it is based on a single individual, namely a dictator. For a nondictatorial process f, the relationship between u, s, and the behavior of $f(u,s)$ can be extremely difficult to analyze.

The Peixoto–Smale analysis suggests one way to classify f. Let *Chaos* be the domain in the space $U\times S$ where the dynamical system $f(u,s)$ is chaotic. Similarly, let *Stability* be the domain in $U\times S$ where $f(u,s)$ is structurally stable.[3] Smale's result suggests that, for an interesting (nondictatorial) aggregational rule, the domain *Stability* cannot be a dense set, at least when the state space has high enough dimension.

One possibility is that chaos itself is generic for f. In this case the geometric structure of *Chaos* and *Instability* can be extremely complex. To illustrate the possible degree of complexity, consider the set of irrationals in a compact interval. This set is a *residual* set (namely a countable intersection of open dense sets) which is thus dense. The complement of this set consists of the irrationals. When chaos is generic, then the domains *Chaos* and *Stability* are analogous to the irrationals and rationals, respectively. Generic chaos can occur, for example, if the aggregation rule

[3] As implied by note 2, there is also a domain (*Instability*) in $U\times S$ where $f(u,s)$ is structurally unstable.

is based on a voting mechanism of a certain type.[4] A second possibility is that both *Chaos* and *Stability* contain open subsets (and so have nonempty interiors) but neither one is dense.

One research program that has developed from Arrow's result has implications for an understanding of economic processes and suggests that a particular kind of chaos resulting from "manipulation" can be associated with an open class of economies. Gibbard (1973) and Satterthwaite (1975) both gave a significant interpretation of Arrow's theorem when they showed that, with a nondictatorial social choice rule $f(u)$ (that specified a chosen state), individuals may misrepresent their preferences in an attempt to achieve a more preferred outcome. Hurwicz (1972) noted the connection of Arrow's theorem to the analysis of economic systems by demonstrating that exchange economies were susceptible to formal manipulation of this kind. Many authors since then have examined manipulation in preference, or in endowments (Aumann and Peleg 1974; Gale 1974; Guesnerie and Laffont 1978). An important result by Safra (1983) showed that the set of economies that are susceptible to preference manipulation (and therefore resource manipulation) are "rich." More precisely, if we consider the set of all economies, parameterized in some fashion and endowed with an appropriate topology, then there exist open sets of both manipulable and nonmanipulable economies. In a manipulable economy, at least one individual (or agent) is capable of misrepresenting preferencs or resources (perhaps by treating prices nonparametrically) to affect the way the economy behaves in an attempt to bring about a more preferred outcome. Common sense might tell us that the impact of one individual on an economy must be negligible, so that the practical consequences of these results are unimportant. Such an inference is not entirely self-evident. At the time of this writing (September 1992), it is possible that foreign exchange markets – now being described by such terms as "mayhem" and "disorder" – are out of equilibrium, but it is also likely that the markets are engaged in intense speculative manipulation, triggered by political uncertainty in Europe and the United States. A plausible inference is that a manipulable economy will exhibit chaos in its development path. (See a number of the chapters in the volume edited by Barnett, Geweke, and Shell 1989 for theoretical and empirical analyses of speculative bubbles, sunspot equilibria, and complex dynamics.)

4 For a voting procedure, we may ignore the space S. The appropriate parameter space U of smooth profiles is endowed with the Whitney topology. With this topology U is a Baire space. For a voting rule f, the domain *Chaos*, under some conditions, can be shown to be residual and thus dense (McKelvey and Schofield 1986). A *generic property* is one that is satisfied on a dense, residual set.

The results briefly mentioned here on economic and political manipulation are strictly valid only for a simple aggregation procedure, defined by first-order variables such as the profile u and parameter s. To make the social behavioral rule f more realistic, one may add second-order characteristics by modeling in more detail the calculus and behavior of the individuals. While the natural vehicle for any such modeling attempt is game theory, there are a number of ways in which game theory and social choice theory can be combined to give a more complex account of individual behavior.

One way of dealing with the problem of manipulation is through the concept of *implementation*. Given an aggregation procedure f, can a social planner construct a compatible game form (namely, a list of strategy sets for the individuals and a system of regulations or rules) that "oblige" rational individuals to reveal, in some sense, their true preferences and characteristics? As Groves and Ledyard (1980) showed, it is possible to construct a game form that requires individuals to truly reveal their characteristics in Nash equilibrium. As Hurwicz (Chapter 2 in this volume) observes, however, why should individuals restrict themselves to the strategy sets specified by the game form? It may be necessary to add enforcers to monitor behavior. These become agents within the game itself, who themselves require monitoring.

A second possibility with regard to the addition of structure to f is to incorporate beliefs of the individuals. In the manipulation literature the focus is on the existence of an individual i, say, who may lie about u_i or s_i to change the outcome. However, it is clear that there must be an information requirement governing the belief, by i, that such a switch would be beneficial. Suppose i has information that a second individual, j, may also manipulate and bring about an outcome which is worse for both agents. This situation has the structure of a two-person Prisoner's Dilemma. In the single-shot game, each agent has a dominant strategy of noncooperation, so the resulting Hobbesian outcome is non-Paretian. If both cooperate, they can attain a Pareto-optimal outcome. If we consider the extended game, involving n participants, as a simple aggregation procedure, then almost anything can happen (Schofield 1977). Although cooperative coalitions can come into being, they can just as easily collapse. Reiterating the game in time does little to diminish the degree of chaos: Although cooperative coalitions can occur in equilibrium (Taylor 1976), their maintenance depends on knife-edge calculations that ultimately are based on individual beliefs concerning other individuals' behavior. As Calvert shows (Chapter 8), it is possible to sustain cooperation in the two-person game under certain "belief scenarios." However, these beliefs fundamentally depend on foundations in common knowledge (A believes

B believes A . . . will cooperate). As Kreps et al. (1982) have demonstrated, these belief structures may become unstable if the game nears a point of termination.

A more general procedure is to combine these two approaches by adding institutional structure to the aggregation procedure f. That is, we may incorporate complex rules to restrict the behavior and beliefs that individuals use to make sense of their choices. Even institutional enforcement cannot completely maintain institutional rules and regulations, so these will change with time. The beliefs of members will also slowly change to accommodate changing behavior and environment. Underlying the institutional approach is the deep problem that game theory, social choice, and political economy are all obliged to face in attempting to model rational behavior. How exactly do individuals form beliefs about the world and about the behavior of others? Is the process of belief formation (and the consequent individual action) structurally stable? If it is not, then imperceptible perturbations in the structure of any institution can lead to dramatic changes in behavior. This gives a justification for North's view (Chapter 3) that institutional evolution is likely to be "path-dependent." On the other hand, if beliefs were solely based on the operation of the natural world, then one might expect convergence to consensus (McKelvey and Page 1986), at least when communication is rich enough (Weyers 1992). However, if beliefs are held not only about the natural world but also about human rights and responsibilities (as we must expect), then ideological differences could be irreconcilable (see Hinich and Munger, Chapter 1) and human behavior fundamentally chaotic.

The debate concerning the possibility of chaos has been quite vigorous in formal political theory (see the discussion in Ordeshook, Chapter 4). It seems more or less self-evident that political systems can break down unexpectedly into a state of chaos or near-chaos. Aside from noting the examples of the Soviet Union, Eastern Europe, Yugoslavia, and Lebanon, one may wonder about the degree of political stability (as of September 1992) in Italy, for example, or with regard to the process of European integration.

In the analysis of voting models, Arrow's theorem (1951) is often construed in terms of the possibility of constructing a voting cycle (x is socially preferred to y is socially preferred to z . . . is socially preferred to x). Although existence of such a cycle by no means implies chaos, it does suggest that the social choice rule is not "first-order," but rather must be based on additional features of the decision-making institution. Early attempts to circumvent Arrow's theorem showed that cycles could be eliminated if the preference profile were restricted in some fashion (Sen 1970), or if the number of alternatives were sufficiently limited (Ferejohn and

Grether 1974; Nakamura 1979). If the set of alternatives W is geometric in form and restricted to one dimension (and compact), then it was shown that a *core* (or unbeaten point) exists under majority rule (Black 1958; Downs 1957). Interpreting this result in terms of committee decision making or direct democracy implies that the process is structurally stable, since this core acts as a stable global attractor.

There are problems with the direct application of this result to two-party representative democracy. If the parties are uninterested in policy per se, but only concerned with winning, then they should converge on the core point in choosing policies or manifestos to present to the electorate. However, it seems unlikely that this is an appropriate model for political competition; candidates in a two-party system generally do not converge. It was suggested by Wittman (1977, 1983) that if parties had their own preferred policy points then they would not converge in their declared positions. There are two difficulties with this proposition. First of all, if candidates are certain about the electoral response to the competing candidate positions, then any candidate who adopts a compromise (noncore) position will be beaten by an opponent who chooses the core. As a means of implementing a preferred policy point, such a strategy is irrational (at least on the face of it). To maintain divergent positions of the candidates, it is necessary to modify the model in some way. The usual procedure is to smooth the electoral response by assigning probabilities, say ρ_1, ρ_2, that candidates 1 or 2 win when their positions are z_1, z_2, and to show existence of a Nash equilibrium in these positions (Cox 1984, for example). A second difficulty arises in this case. Having won with a position z_i, there is no reason for party i to implement this declared position rather than its true position (Wittman 1990).

This problem of credible commitment by parties to their declared positions is essentially one of a "belief equilibrium" by both parties and voters. If parties (or candidates) can be punished, in some way, for violating their promises, then it is intuitively obvious that the candidates can be bound to their promises. A number of the chapters in Section IV of this volume show the existence of belief and behavioral equilibria in two-party models of competition. Although the models are based on a 1-dimensional space, they show, in the terminology introduced herein, that the aggregation mechanism will be structurally stable when it incorporates second-order phenomena of this kind.

In contrast to the 1-dimensional case, a simple, first-order aggregation procedure based on majority voting in two dimensions can be chaotic. More precisely, McKelvey (1976) showed that if the core is empty then, under *simple* majority rule $f(u)$, it is possible to construct discontinuous voting trajectories that go almost everywhere in the policy space W. Extensive experimental work (reviewed in McKelvey and Ordeshook 1990) indicated that voting mechanisms in two dimensions were generally Paretian and thus not completely chaotic.

A possible explanation is obtained by requiring $f(u)$ to be *continuous,* in the sense that only continuous voting trajectories defined by $f(u)$ are considered. Then the cycles that are generated must be restricted to the Pareto set in two dimensions (Schofield 1978). However, the *voting classification theorem* (Schofield 1984; McKelvey and Schofield 1986) essentially showed that continuous voting trajectories under majority rule, $f(u)$, can certainly wander almost anywhere in four dimensions. The full result classifies any nondictatorial and continuous voting method f by two integers: the stability dimension $v(f)$ and the instability dimension $w(f)$. If the dimension w of the space W is no greater than $v(f)$, then a core exists for any profile of convex preferences. If w is at least $w(f)$ then the core is generically empty, while if $w > w(f)$ then f is generically chaotic.[5] For majority decision making, $v(f) = 1$ whereas $w(f) = 2$ or 3, depending on whether the size of the electorate is odd or even.[6]

A response to this classification result is to infer that direct democracy can almost never define a social welfare function of the Arrovian kind and to seek instead those conditions that are likely to enhance or restrict the degree of political manipulation (Riker 1982, 1986). As we have noted, one way to proceed is by incorporating second-order institutional features that constrain political choice (Shepsle 1979; Shepsle and Weingast 1981) and create equilibrium. A related idea is to construct a "prediction set" – such as the *uncovered set* (Miller 1980) or *yolk* (McKelvey 1986) – which is defined by the second-order characteristics of the simple voting mechanism $f(u)$ and which lies inside the Pareto set. Particular kinds of institutional rules, called amendment procedures, can lead sophisticated voters to behave in ways that give outcomes in the uncovered set. Indeed, such procedures can force voters to truthfully reveal their preferences (Groseclose and Krehbiel, Chapter 10).

Underlying the classification theorem is an Arrovian-like unrestricted domain assumption that all smooth profiles are permitted. One can modify

[5] For a smooth profile u and rule f, we say $f(u)$ is *chaotic* if a continuous voting trajectory can be constructed between almost all points in the space W. As observed in note 4, a generic property of f is one that holds for all u in a residual subset of U. Such a subset will be dense and, in the case of a compact space W, also open. Thus *Chaos* for a voting rule can be even more pervasive than for a dynamical system or exchange economy.

[6] Thus chaos is generic for majority rule in three dimensions (for n odd) or four dimensions (for n even). For any democratic voting rule f, it is possible to compute $w(f)$, at least in principle.

this assumption and examine the connecton between symmetry assumptions on the distribution of voter preferences and $v(f)$. For example, if the distribution is log-concave and the voting rule requires a 64-percent majority, then the rule possesses a core (i.e., an α-core) irrespective of the dimension (Caplin and Nalebuff 1988). Schofield and Tovey (1992) have analyzed the connection between the classification theorem and these results on the α-core, and Tovey (Chapter 7) explores a number of consistency and computational problems associated with such social choice estimators as the α-core and yolk.

The classification theorem is certainly valid for a simple voting mechanism f, such as a direct democracy without second-order institutional features. For a representative democracy, chaos, in some sense, can occur if (i) there are only two parties or candidates; (ii) there is certainty in electoral response to candidate messages or positions; and (iii) the dimension of the policy space is at least $w(f)$. Even if candidates have policy objectives, for any position z_i by candidate i there is a position z_j by j that beats z_i and which i prefers. However, a *Nash* equilibrium in candidate positions will exist under general conditions (Cox 1987, 1989; Coughlin and Nitzan 1981; Enelow and Hinich 1984) as long as there is electoral uncertainty, no matter what the dimension. The problem of credible commitment remains, however, and may be difficult to solve.

Models of *multiparty* competition (i.e., with at least three parties), with or without electoral uncertainty, are much more difficult to construct than two-party models. If the candidates attempt to maximize the number of seats they control, then – as Eaton and Lipsey (1975) conjectured and Shaked (1975) showed – there is no equilibrium in the choice of positions. Incorporating policy preferences for parties brings in the problem of credible commitment again. Moreover, the motivational basis for a pure vote-maximization model is obscure. If parties are concerned with policy, then it makes more sense to model the effect of their strategy on the eventual policy outcome.

Schofield (Chapter 6) constructs an equilibrium model in arbitrary dimension where many parties calculate what policy to declare to the electorate by reference both to electoral concerns and final government policy. To do this, however, it is assumed that parties have consistent beliefs concerning the nature of interparty negotiation. It is possible that many of the results (in section IV of this volume) on two-party political competition and the underlying structure of party–voter beliefs can be extended to the general case of many parties and arbitrary dimension. Some insights into how governments may acquire expert or voter information relevant to decision making in multiple dimensions are given in Section V.

2 An overview of the essays

The previous section provided a general framework for interpreting recent work in political economy presented in this volume. In organizing the various contributions, the aim has been to bring out certain themes that run throughout the earlier discussion. Part I, "Perspectives in Political Economy," focuses on some of the general themes: the nature of ideology, institutions, incentives, and the balance between theoretical and empirical work in this relatively new discipline. Part II, "Representation and Voting," deals first with the axiomatic foundation of representation; the next two chapters discuss general voting procedures as methods of preference aggregation on a policy space of unrestricted dimension.

Part III, "Political Institutions," considers the different characteristics that govern the behavior of an institution, conceived in general terms. Three of the chapters focus on communication, formal procedural rules in voting, and on seniority, while a fourth chapter uses the concept of credible commitment to interpret the events leading up to the Civil War in the United States. Part IV is devoted to modeling political competition in a 1-dimensional policy space. The models incorporate candidate abilities, incomplete information, expenditures, and so forth. Part V is concerned with the way in which government may acquire or aggregate information held by voters or advisers. Part VI, comprising the final two chapters of the volume, deals with government choice on monetary policy and taxation.

2.1 *Part I: Perspectives on political economy*

In "Political Ideology, Communication, and Community," Hinich and Munger address the related concepts of *community* and *ideology*. Although the authors do not spell out the meaning of ideology quite so generally, one can think of ideology as a general system of beliefs about the way the world works, and by inference, the way other people think as well. Implicit is the conception of rights of individuals and collectivities. Hinich and Munger are aware of the difficulties of the origin of ideology, but argue that ideology itself is stable. An immediate consequence of their argument is a change in the way we might study political competition. As they say, "it is wiser to find out what a candidate believes on all issues than it is to decide whether to believe what she says issue by issue."

Another key concept in political economy is the idea of *implementation:* the construction of a game form which has the effect of forcing