Index

Admissible triple \((r, s, n)\), 10, 125, 137, 169
Approximation lemma, 258
Artin’s theorem connected with Cassels’ lemma, 15
Artin’s theorem on Hilbert’s 17th problem, 46
Aubry’s lemma (see Davenport-Cassels lemma), 29, 30
Bezout’s theorem, 103
Cage of a form, 96
Calderon’s theorem, 113
Cassels’ lemma, 15
Cassels-Pfister lemma, 150
Cayley numbers (octonions), 2, 138, 139
Choi’s theorem, 109
Choi-Lam proof of Hilbert’s theorem, 85
Choi-Lam-Reznick theorem, 100
Davenport-Cassels lemma (see Aubry’s lemma), 29, 30
Dubois’ example, 56
Eight square identity, 2
Euler identity, 1
Extremal form, 82
Exact sequence, 237
Field
Formally real, 211
Global, 269
Local, 269
Ordered, 210
of formal power series, 57, 269
Pythagorean number of \(a\) (or reduced height of \(a\)), 44
Pythagorean, 230
Pythagoras closure of \(a\), 231
Quadratically closed, 230
Real closed, 216
Real closure of \(a\), 222
Stufe of \(a\), 13
Form, 72
Four square identity, 1
Gabel’s theorem, 198
Hilbert’s 17th problem, 46
Hilbert’s theorem in \( \mathbb{R}[X_1, \ldots, X_n]\), 72, 85
Hilbert’s Nullstellensatz, 92
Hilbert-Landau theorem, 224
Hilbert-Siegel theorem, 260
Hopf condition, 169
Hsia-Johnson-Pourchet theorem, 265
Hurwitz matrix equations, 127
Hurwitz theorem on bilinear identities, 3
Hurwitz-Radon theorem, 127
Hurwitz-Radon (HR) family, 208
Hyperbolic forms, 147
Kirkman identity, 195
Krein-Milman theorem, 88, 277
Kronecker product of matrices (see tensor product)
Lagrange’s interpolation formula, 81
Landau’s theorem in \(\mathbb{Q}(X)\), 47
Legendre’s theorem, 155
Level (= Stufe), 13
Map
Bilinear, 187
Biskew, 187
Non-singular, 187
Normed, 187
Index

286

Motzkin ternary sextic, 47, 73

Octonions (see Cayley numbers), 2, 138, 139

Pfister's theorem on

- square identity, 12, 19, 20
- Stufe of fields, 13
- Pythagoras number of function fields, 47, 60
- Pfister forms, 63, 159
- Positive-semi-definite (PSD) forms, 31, 72
- Prouhet's theorem, 241
- Pythagoras number of fields, 44, 227
- of function fields, 47
- of algebraic number fields, 266

Quadratic form

- Anisotropic, 143
- Bilinear form of a, 66
- Binary, 148
- Determinant of a, 142
- Diagonal, 144
- Direct sum of, 144
- Equivalent, 142
- Isotropic, 143
- Matrix of a, 142
- Multiplicative, 163
- Non-singular, 142
- Strongly multiplicative, 166
- Singular, 142
- Universal, 146
- Quaternions, 139

Radon function, 127

Radon's theorem, 127

Rational admissibility, 106

Reduced height, 227

Representatives, 142

Representation theorem

- first, 143
- second, 151

third, 153

Reznick's theorem on

- simplest forms, 94

Robinson's

- even symmetric ternary sextic, 119, 121
- ternary sextic, 76
- theorem, 116
- quaternary quartic, 75
- Round form, 161

Shapiro's theorem, 182

Shapiro-Leep theorem on the

- Stufe of cyclotomic fields, 34
- Similarity factor, 24, 154
- Sixteen square problem, 126

Springer's theorem, 42

Square classes, 26, 228

Stufe, vii, 13

Stufe of

- biquadratic fields, 44
- cyclotomic fields, 33, 260, 263
- fields, 13
- finite fields, 33
- rings, 70
- quadratic fields, 31
- Sums of squares (SOS), 29, 72

Tensor product (Kronecker product)

- of matrices, 139
- Totally complex algebraic number field, 40
- Totally positive element, 214

- Tsen-Lang theorem, 67

Universal (quadratic form), 146

Van der Waerden's problem, 27

Witt's cancellation law, 145

Zero set of a form, 76, 77