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978-0-521-42566-7 - Stability, Instability and Chaos: An Introduction to the Theory of
Nonlinear Differential Equations

Paul Glendinning

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