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London Mathematical Society Lecture Note Series, 162

Harmonic Analysis and Representation Theory for Groups Acting on Homogeneous Trees

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More information

CONTENTS

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P R E F A C E

Over the past few years, we ran a Seminar in Harmonic Analysis at the Mathematics Department of the University of Rome "La Sapienza". In this seminar many of the talks given by staff members and visitors were concerned, directly or indirectly, with infinite trees or tree-like graphs, and their automorphism groups. Seminar notes were occasionally taken by one or both of us, and sometimes written up informally for distribution to newcomers to the seminar. After a while, we felt that it would be convenient to give a more coherent organization to these notes. Once this decision was taken it became apparent that, at the cost of some omission, the general aim of describing the group of automorphisms of a homogeneous tree and its irreducible unitary representations would provide a convenient focus which would include much of the material we had in mind. We felt that this approach would shed light on the connection between harmonic analysis on trees and harmonic analysis on hyperbolic spaces, by emphasizing the strict analogy between the group of automorphisms of the tree and real rank 1 semisimple Lie groups. This choice left out a lot of valuable material specifically concerning free groups and free products of finite groups. We felt however that the notes **[F-T P2]** and the memoir **[F-T S2]** could provide an introduction to these topics. We also decided not to treat the case of a semihomogeneous tree. Semihomogeneous trees are natural objects because they are exactly the Bruhat-Tits buildings of rank 1 **[BT]**. It follows that every rank 1 reductive algebraic group over a local field is a closed subgroup of the group of automorphisms of a semihomogeneous tree. We felt however that, while no major conceptual step is needed to extend the theory of representations of the group of automorphisms of a homogeneous tree to the case of a semihomogeneous tree, from a practical point of view the notation would have become more burdensome. The connection with matrix groups over local fields is explained in these notes by giving, in the Appendix, Serre's construction of the tree of **PGL**(2,$\mathfrak{F}$) where $\mathfrak{F}$ is a local field.

Chapter I contains a description of the geometry of a homogeneous tree $\mathfrak{X}$ and its boundary, the group of automorphisms Aut($\mathfrak{X}$) and some of its notable subgroups. Chapter II contains the boundary theory for eigenfunctions of the Laplace (or Hecke) operator on the tree and a complete description of spherical functions and spherical representations which applies to every closed subgroup of Aut($\mathfrak{X}$) with transitive action on the vertices and the boundary of the tree. It also contains the proof of an important result due to T. Steger which asserts that every spherical representation (with one possible exception) of Aut($\mathfrak{X}$) restricts irreducibly to any cocompact discrete subgroup. Chapter III contains a description of square-integrable representations of Aut($\mathfrak{X}$) following the beautiful geometric classification due to G.I. Ol'shianskii. At the end of the chapter we give the complete Plancherel formula for Aut($\mathfrak{X}$).

The Appendix, as already mentioned, contains a complete and elementary account of the construction of the tree of **PGL**(2,$\mathfrak{F}$) and a discussion of the action of this group on its tree.

These notes owe a great deal to the many friends, colleagues and students who participated in our seminar. We would like to thank especially M.G. Cowling, F.I. Mautner and R. Szwarc for the many critical observations and comments which stimulated our work and often found their way into these notes.

Very special thanks are due to Tim Steger who contributed in many ways with help and advice in the preparation of these notes. He also gave his permission to include in these notes his yet unpublished restriction theorem, which was presented at our seminar in 1987.