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Representations of Algebras and Related Topics

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Contents

INTRODUCTION

The Tsukuba International Conference on Representations of Algebras and Related Topics[1] took place in the week before the International Congress of Mathematicians in Kyoto (1990). The Conference was preceded by a Workshop in which leading workers in the field gave expository lectures on recent developments in areas covered by the Conference.

The aim of this book is to present the Workshop lectures to a wider audience. The participants at the Workshop were not all specialists in the area and so the speakers aimed to make their talks as self-contained as possible. This is reflected in their papers presented here. Several of the authors have taken the opportunity to update their manuscripts, most of which contain results which have not appeared elsewhere. We have included one paper (Dlab's) which was not presented at the Workshop.

The Tsukuba meetings took place at a time of exciting and highly complex interaction between the representation theory of algebras and other branches of mathematics. Several of the powerful technologies developed within algebra representation theory during its rather introspective period from about 1970 to the mid-1980s are now contributing strongly to other areas. In the opposite direction, new problems, ideas and points of view are coming into the subject from previously unrelated areas. Some of these interactions are reflected in the present volume.

The study of functor categories (categories of abelian group valued functors on module categories) is one of the most successful methodologies of algebra representation theory and underlies key concepts like those of almost split sequence and Auslander-Reiten quiver. The homologically finite subcategories discussed in the paper by Auslander and Reiten are subcategories of modules for which the operation of restriction of finitely presented functors is especially well behaved and, for example, can imply the existence of relative almost split sequences. In their paper Auslander and Reiten show that there is an intimate relation between homologically finite subcategories and the generalised tilting modules of Happel and Miyashita. Tilting modules have been generalised still further to tilting complexes in Rickard's Morita theory for derived categories, and generalised tilting theory has opened the way for the importation to algebra representation theory of ideas and methods from, for example, the theory of vector bundles.

Derived categories do not appear explicitly in this volume. However the attractive and widely occurring class of quasi-hereditary algebras, discussed here by Dlab and Ringel, had its origins in the use of derived categories and tilting modules for the study of representations of semi-simple algebraic groups in characteristic $p > 0$.

Another important methodology has emerged from the study of

matrix methods and their formalisation in the theory of bocses, and is the basis of the only proof of the tame-wild dichotomy for algebras over an algebraically closed field (Drozd's Theorem). In his present paper Drozd describes some recent advances in bocs theory and a new application of it to obtain a description of dense sets of irreducible unitary representations of certain Lie groups of mixed type.

Although the tame-wild dichotomy applies only to finite dimensional modules, the essential definitions use infinite dimensional modules. Crawley-Boevey has used indecomposable modules of infinite length and finite endolength (length over the endomorphism ring) to define concepts of generic tameness and wildness which may provide the basis for a tame-wild dichotomy for wider classes of rings. Progress in this direction is amongst the exciting results contained in his paper in this volume. In it he uses both functor and matrix methods in new and wider contexts.

The rings of pure global dimension zero of Azumaya's article include the rings of finite representation type. Such rings R have been characterised by a set of equivalent conditions which must be satisfied by all R-modules. Azumaya shows that, in each case, it is sufficient to require that the condition be satisfied by all countably generated R-modules.

The use of quivers, in several forms, has become characteristic of algebra representation theory and provides a point of contact with many other branches of mathematics. One notable example is Kac's use of invariant theory to describe the representation theory of quivers in terms of Kac-Moody root systems. A direct connection between representations of a Dynkin quiver and the positive part, U_+, of the universal enveloping algebra of the corresponding Lie algebra was established by Ringel and subsequently extended by him to the 'quantum' situation. This has stimulated other work on the relationship between representations of finite dimensional algebras and Lie algebras and the use of quiver representations in the study of quantum groups. In his present paper Ringel shows how the coalgebra structure of U_+ may be understood in terms of his construction from representations of the corresponding quiver.

Dlab's paper also uses representations of quivers and, more generally, species. He discusses a problem originating in the work of Jones on type II_1 factors which led to the Jones polynomial in knot theory. Values of the Jones index which Dlab derives here using quiver representations are also the singular values of a parameter which occurs in knot theory and quantum groups.

Auslander-Reiten quivers impose an extremely useful geometric structure on the categories of finitely generated modules over an artin algebra. In her study of tame blocks for group algebras, Erdmann uses, with consummate skill, the interaction between this geometric structure and the special properties of the blocks. Her paper describes this work and shows how such methods may be used in somewhat more general situations.

Carlson's paper is concerned with the cohomology theory of finite groups. His aim is to find a zero point for indexing the doubly infinite sequence of Tate cohomology. He defines a new invariant of a module, which may serve this end. His paper includes extremely useful reviews of a wide range of topics. The final section of Carlson's paper is independent of the earlier part. In it he discusses two questions concerning contravariantly finite subcategories which were raised at the Workshop.

The Workshop received substantial support from both the Exchange Fund of the University of Tsukuba and the Hara Research Fund. We are most grateful for their assistance. We also wish to express our thanks to the authors and referees of the papers in this volume and to David Tranah of the Cambridge University Press for his help in preparing it for publication.

Sheila Brenner Hiroyuki Tachikawa

[1]The Proceedings of the Conference will be published separately as Canadian Mathematical Society Proceedings series No. 11.