ACCADEMIA NAZIONALE DEI LINCEI

SCUOLA NORMALE SUPERIORE

Lezioni Fermiane

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The Theory of Singularities and Its Applications

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PREFACE

The mathematical description of the world depends on a delicate interplay between discrete and continuous objects. Discrete phenomena are perceived first, but continuous ones have a simpler description in the terms of the traditional calculus. Singularity theory describes the birth of discrete objects from smooth, continuous sources.

The main lesson of singularity theory is that, while the diversity of general possibilities is enormous, in most cases only some standard phenomena occur. It is possible and useful to study these standard phenomena once for all times and then to recognize them as the elements of more complicated phenomena, which are combinations of those standard elements.

In these lectures I shall describe some of such elementary singularities and some of the simplest of their applications. For instance, we shall consider the singularities of the domains in functional spaces, defined by such conditions, as the conditions of stability, of ellipticity, of hyperbolicity, of disconjugacy and so on, and the unexpected relations of singularity theory to reflection group theory (where, for instance, the variational problem of the by-passing of an obstacle, bounded by a smooth plane curve or space surface, is related to the icosahedron or to the hypericosahedron, that is to the regular-600-hedron in 4-space).

The results of singularity theory may be understood and used independently of their proofs, which are rather technical and involved. The proofs of the majority of the results, discussed below, may be found in the book "Singularities of Differentiable Mappings" by V.I. Arnold, S.M. Gusein-Zade, A.N. Varchenko, Moscow, Nauka, vol. 1, 1982, vol. 2, 1984 (English translation by Birkhäuser, vol. 1, 1985, vol. 2, 1988) and in the series Itogi Nauki i Techniki, Sovremennye Problemy Math., Novejshie Dostijenia, Moscow, Viniti, vol. 22 (1983) and vol. 33 (1988) (translated into English as J. Soviet Math. by Consultants Bureaux, Plenum, N.Y.; vol. 22 is translated in 1984 as part of the vol. 27 of the J. Sov. Math.). For more details on singularities, see the survey "Singularities" by V.I. Arnold, V.A. Vassiljev, V.V. Gorjunov and O.V. Ljashko in the series Itogi Nauki i Techniki, Sovremennye Problemy Math., Fundamentalnye napravlenia, Moscow, Viniti, vol. 6 (1988) and vol. 39 (1989) (translated by Springer as "Encyclopedia of Mathematical Sciences", Dynamical Systems volumes 6 and 8).