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# Categories and Computer Science

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In memory of my father
Lynn David Walters

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# Preface

This book has its origins in an undergraduate course I have given on Categories and Computer Science over the past three years to a total of approximately 200 students at the University of Sydney. The course is a gentle introduction to category theory, with motivating examples taken from computer science, and was developed through interaction with the students.

Although this book is an undergraduate text, it contains quite a deal of original material, further details of which may be found in my reports and papers listed with the references. In particular, much of the discussion of distributive categories, data types, and imperative programs is new, and Chapter 7, §2 contains a description of a generalization of the Todd–Coxeter algorithm suitable for computing left Kan extensions developed with Sean Carmody; this section was written together with Sean.

The idea for the course came from discussions with Bill Lawvere while he was in Sydney in 1988. Since then we have held in Sydney, in addition to the Sydney Category Seminar (now 21 years old, chaired by Max Kelly), a further seminar — The Sydney Categories in Computer Science Seminar. I have learnt much from the members of this seminar, visitors to the seminar, and from my graduate students, fourth-year students, and vacation scholars. My thanks go to David Benson, Aurelio Carboni, Sean Carmody, Robin Cockett, Robbie Gates, Diana Gibson, Mike Johnson, Lesley Johnston, Giulio Katis, Wafaa Khalil, Stephen Lack, Mark Lauer, Phil Lavers, Mark Leeming, Stephen Ma, Gordon Monro, Wesley Phoa, Steve Schanuel, Usha Sridhar, Sun Shu-Hao, Andrew Solomon, Margaret Thiel, Phil Wadler, and Bill Unger. The research and visitors have been supported by an Australian Research Council Program Grant, and an Australian Research Council Small Grant.

The main assistance in actually producing the book has come from Sean Carmody, Wafaa Khalil, and Karl Wehrhahn. Their help has been invaluable. Additional assistance has been received from Mike Johnson, Stephen Lack, and Wesley Phoa. A further important debt is to my friend and collaborator in Milan, Stefano Kasangian, who first introduced me to the community of computer scientists interested in category theory.

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