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A. A. Ranicki

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102 **Algebraic *L*-Theory and
Topological Manifolds**

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For my parents

SOME ASSEMBLY REQUIRED



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Preface

The algebraic L -theory of quadratic forms relates the topology of manifolds to their homotopy types. This tract provides a reasonably self-contained account of this relationship in dimensions ≥ 5 , which was established over 20 years ago by the Browder–Novikov–Sullivan–Wall surgery theory for compact differentiable and PL manifolds, and extended to topological manifolds by Kirby and Siebenmann.

The term ‘algebraic L -theory’ was coined by Wall, to mean the algebraic K -theory of quadratic forms, alias hermitian K -theory. In the classical theory of quadratic forms the ground ring is a field, or a ring of integers in an algebraic number field, and quadratic forms are classified up to isomorphism. In algebraic L -theory it is necessary to consider quadratic forms over more general rings, but only up to stable isomorphism. In the applications to topology the ground ring is the group ring $\mathbb{Z}[\pi]$ of the fundamental group π of a manifold.

The structure theory of high-dimensional compact differentiable and PL manifolds can be expressed in terms of the combinatorial topology of finite simplicial complexes. By contrast, the structure theory of high-dimensional compact topological manifolds involves deep geometric properties of Euclidean spaces and demands more prerequisites. For example, compare Thom’s proof of the combinatorial invariance of the rational Pontrjagin classes with Novikov’s proof of topological invariance. The current development of the controlled and bounded surgery theory of non-compact manifolds promises a better combinatorial understanding of these foundations, using the algebraic methods of this book and its companion on lower K - and L -theory, Ranicki [146]. The material in Appendix C is an indication of the techniques this will entail.

The book is divided into two parts, called Algebra and Topology. In principle, it is possible to start with the Introduction, and go on to the topology in Part II, referring back to Part I for novel algebraic concepts. The reader does not have to be familiar with the previous texts on surgery theory: Browder [16], Wall [176], Ranicki [143], let alone the research literature*. This book is not a replacement for any of these. Books and papers need not be read in the order in which they were written.

The text was typeset in \TeX , and the diagrams in \LaTeX .

Edinburgh, June 1992

* ‘The literature on this subject is voluminous but mostly makes difficult reading’. This was Watson on integral quadratic forms, but it applies also to surgery theory.