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978-0-521-42023-5 - Asymptotic Behaviour of Solutions of Evolutionary Equations

M. I. Vishik

Frontmatter

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The theme of this book is the investigation of globally asymptotic solutions of evolutionary equations. Locally asymptotic solutions of the Navier-Stokes equations and reaction-diffusion equations are the starting point, and by considering perturbed evolutionary equations, global approximations are constructed.

The lectures upon which this book is based were warmly received at the universities of Rome and Pavia, and at the Scuola Normale Superiore in Pisa. Here Professor Vishik has collated his lecture notes, and has added an appendix describing his recent work on attractors deriving from dynamical systems. This is unquestionably a fine addition to the *Lezioni Lincee*, and will be a necessary addition to the library of all who seek an insight into the solution of evolutionary equations.

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Lezioni Lincee

*Editor: Luigi A. Radicati di Brozolo, Scuola Normale Superiore, Pisa*

This series of books arises from lectures given under the auspices of the Accademia Nazionale dei Lincei through a grant from IBM Italia.

The lectures, given by international authorities, will range on scientific topics from mathematics and physics through to biology and economics. The books are intended for a broad audience of graduate students and faculty members, and are meant to provide a '*mise au point*' for the subject they deal with.

The symbol of the Accademia, the Lynx, is noted for its sharp sightedness; the volumes in the series will be penetrating studies of scientific topics of contemporary interest.

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To the memory of L. A. Lusternik

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