

## INDEX OF NOTATION

 $\|\bullet\|$ , norm, 3 $\mathbb{R}^+$ , non-negative reals, 3 $\mathbb{C}$ , complex numbers, 3 $\mathbb{R}_n$ , real  $n$ -space, 3 $\mathbb{C}_n$ , complex  $n$ -space, 4 $\|\bullet\|_1$ ,  $\ell_1$ -norm, 4 $\|\bullet\|_\infty$ , sup norm, 4 $C[a, b]$ , continuous function on  $[a, b]$ , 4 $(\bullet, \bullet)$ , inner-product, 5 $\ell_2$ , space of square summable sequences, 7 $x \perp y$ , orthogonal, 11 $\omega$ , positive integers, 12 $\sum$ , finite subsets of  $\omega$ , 14 $\ell_2(\Gamma)$ , square summable functions on  $\Gamma$ , 15 $\bar{A}$ , closure, 16 $\text{sp}S$ , linear span of a set  $S$ , 18 $(C[a, b], \|\bullet\|_2)$  continuous functions with  $\|f\|^2 = \int_a^b |f(t)|^2 dt$ , 21 $T : X \rightarrow Y$ , function  $T$  from  $X$  to  $Y$ , 25 $\mathfrak{L}(H)$ , bounded linear operators on Hilbert space, 28 $B(H)$ , bounded linear operators on Hilbert space, 28 $\mathcal{R}(A)$ , range of the operator  $A$ , 28 $M \perp N$ , orthogonal subspaces, 30 $M^\perp$ , orthogonal complement, 30 $M \oplus N$ , direct sum, 30 $Y^*$ , conjugate space, 31 $\ker f$ , kernel of the linear functional  $f$ , 31 $\text{dist}(x, K)$ , distance from a vector to a set, 33 $\sigma(T)$ , spectrum of  $T$ , 36 $\rho(T)$ , resolvent of  $T$ , 36 $\det A$ , determinant, 36

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 $C_r$ , circle of radius  $r$ , 39  
 $r_T$ , spectral radius, 44  
 $\sigma_c(T)$ , continuous spectrum of the operator  $T$ , 47  
 $\sigma_r(T)$  residual spectrum of the operator of  $T$ , 47  
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[More information](#)*Index of Notation* 129 $\alpha_n(T)$ , approximation numbers of the operator  $T$ , 106 $\ell_1(X)$ , operators of type  $\ell_1$ , 106 $\tilde{\text{tr}}$ , extension of the spectral trace, 110 $\beta_{n,p}(T)$ ,  $n$ th  $p$ -mean of  $\{\sigma_i(T)\}$ , 113

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